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**G1G GMB**

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(58) Field of Search
UK CL (Edition N ) **G1G GEL GMB**
INT CL$^6$ **G01V 1/00 1/48 1/50**
**Online: WPI, INSPEC**

## (54) Measurement of nonlinear properties of an earth formation

(57) A method of investigating a formation traversed by a borehole includes, measuring at a plurality of different static borehole pressures the acoustic Stoneley and/or flexural wave velocities of waves propagating through the borehole and formation, and generating an indication of the nonlinearity of the formation by processing the velocity measurements. The velocity measurements are processed either by determining a fractional change in the measured acoustic velocity and dividing that fractional change by the change in borehole pressure to provide frequency dependent acoustoelastic coefficients, or by determining the fractional change in the measured acoustic velocity, and subtracting from the fractional change a component generated by the borehole fluid and a component due to linear aspects of the formation to provide a nonlinear formation component. By processing the velocity measurements at a plurality of frequencies, the nonlinear formation components are used to find nonlinear parameters of the formations. The nonlinear parameters are then used in conjunction with shear wave velocity information and a database of experimental data, to determine the stress in the formation, the strength of the formation, and therefrom, the amount of additional stress required to fracture the



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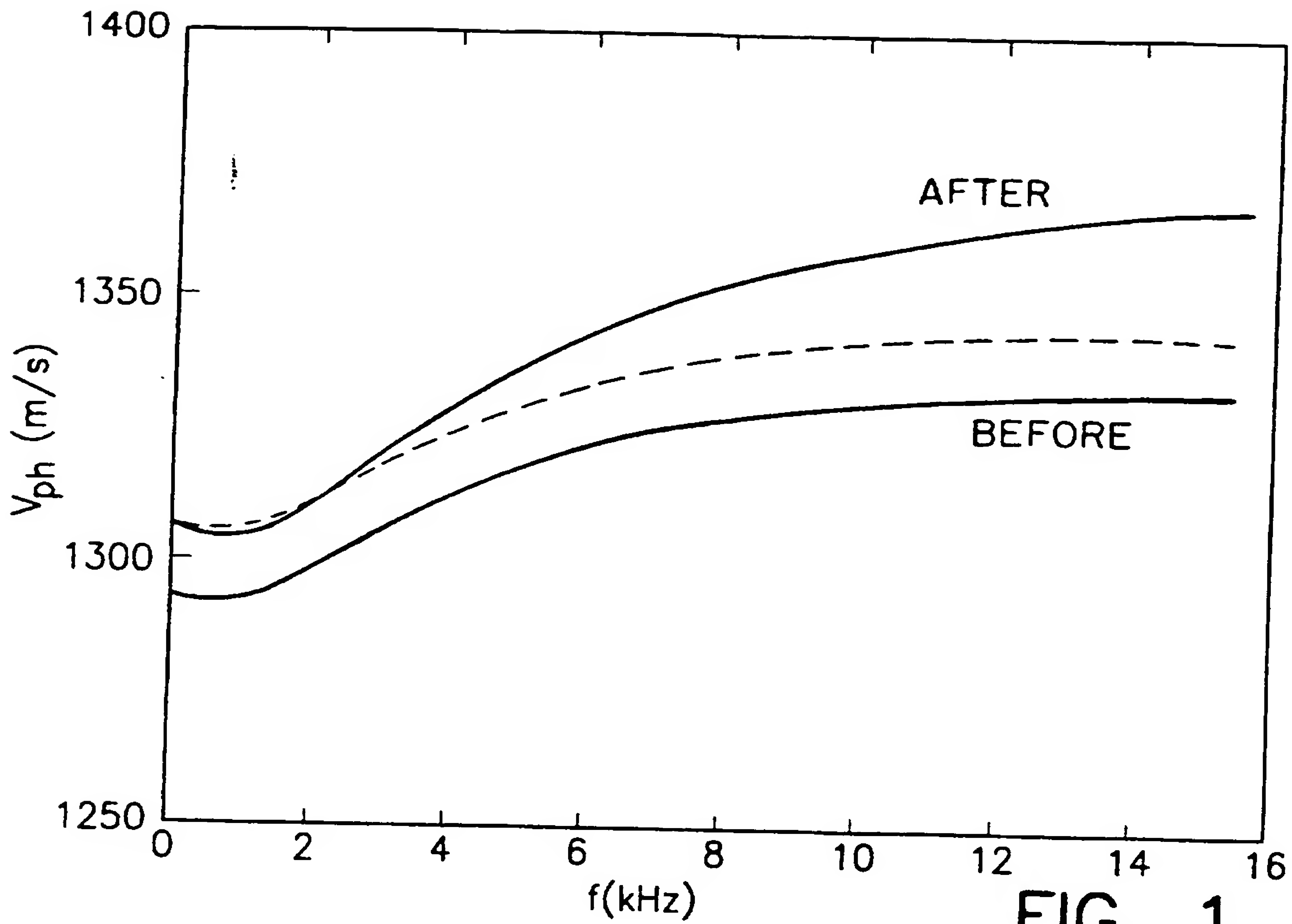
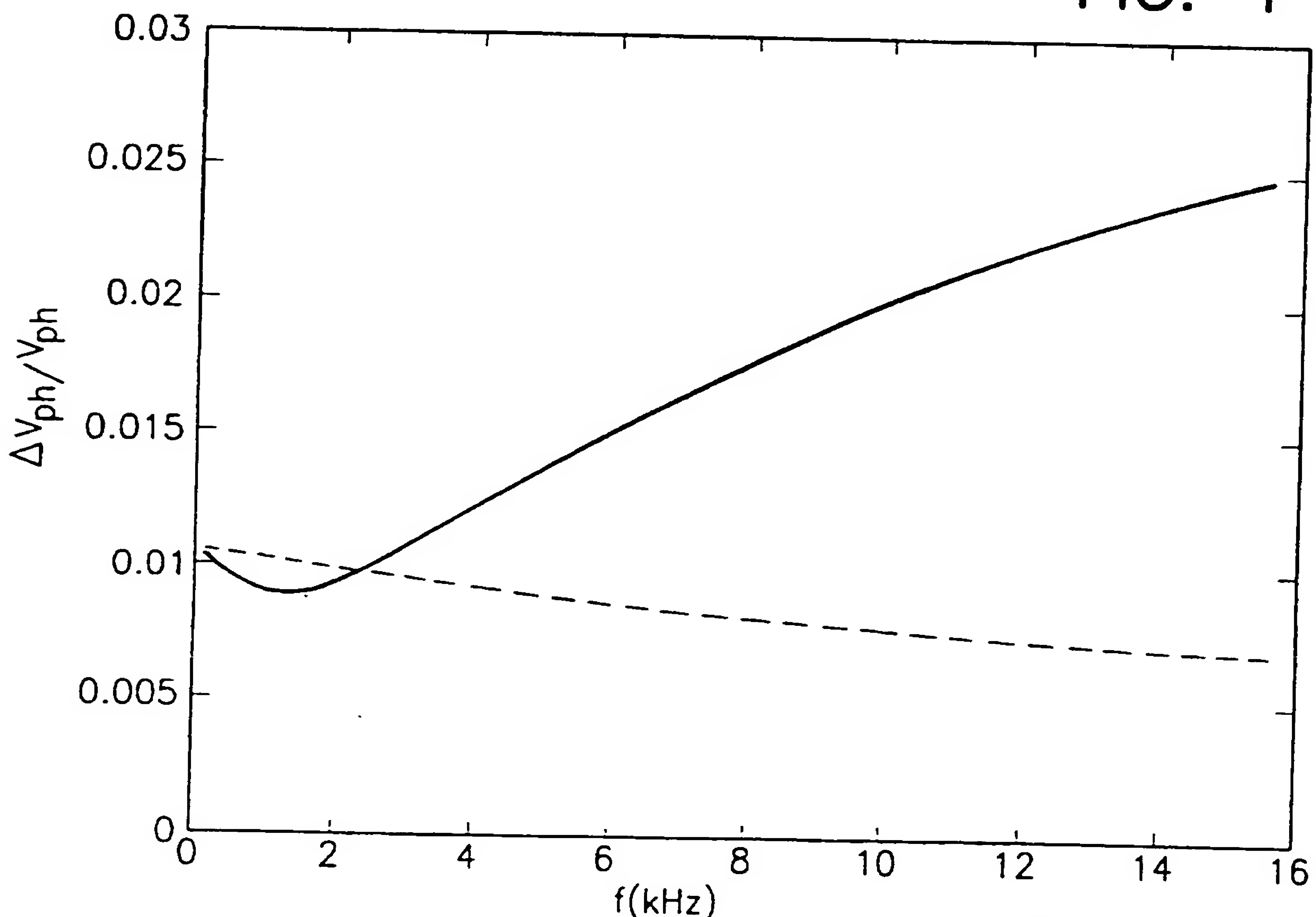


1400
1350
1300
1250
$V_{ph}$ (m/s)
AFTER
BEFORE
0
2
4
6
8
10
12
14
16
f(kHz)


FIG. 1



0.03
0.025
0.02
0.015
0.01
0.005
0
$\Delta V_{ph}/V_{ph}$
0
2
4
6
8
10
12
14
16
f(kHz)


FIG. 2

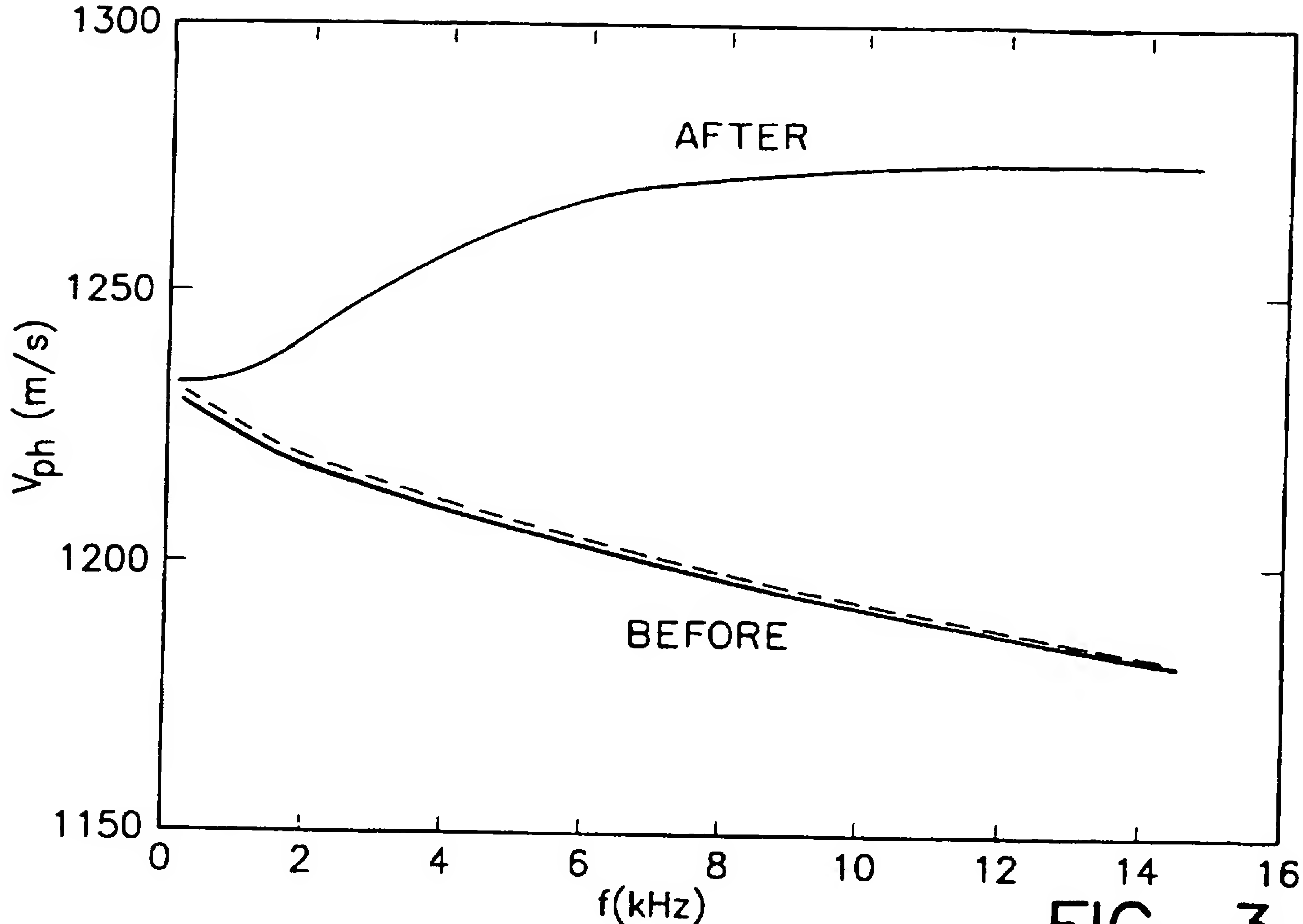
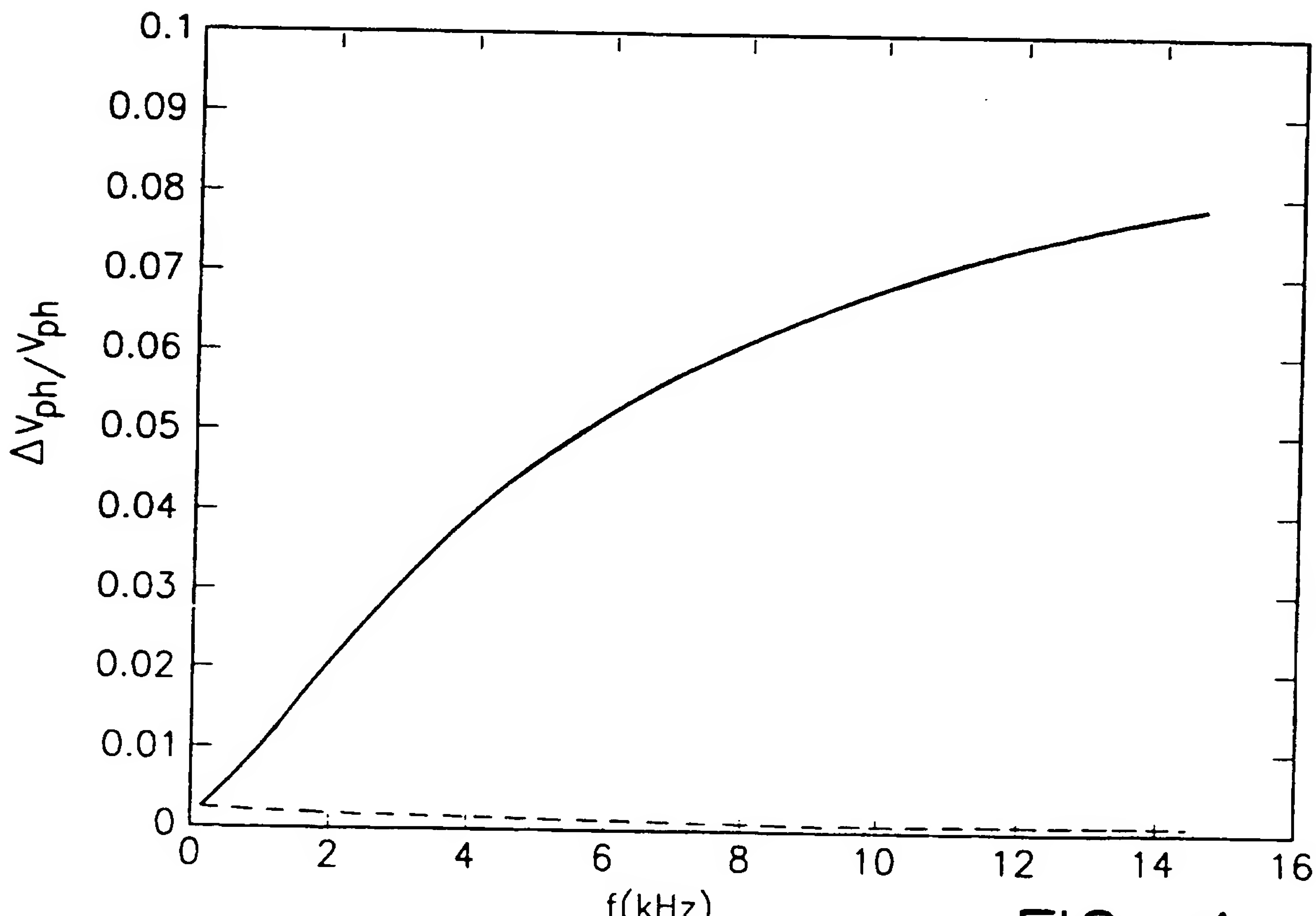

1300
1250
1200
1150
$V_{ph}$ (m/s)
AFTER
BEFORE
0
2
4
6
8
10
12
14
16
f(kHz)

FIG. 3


0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
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0
2
4
6
8
10
12
14
16
f(kHz)

FIG. 4

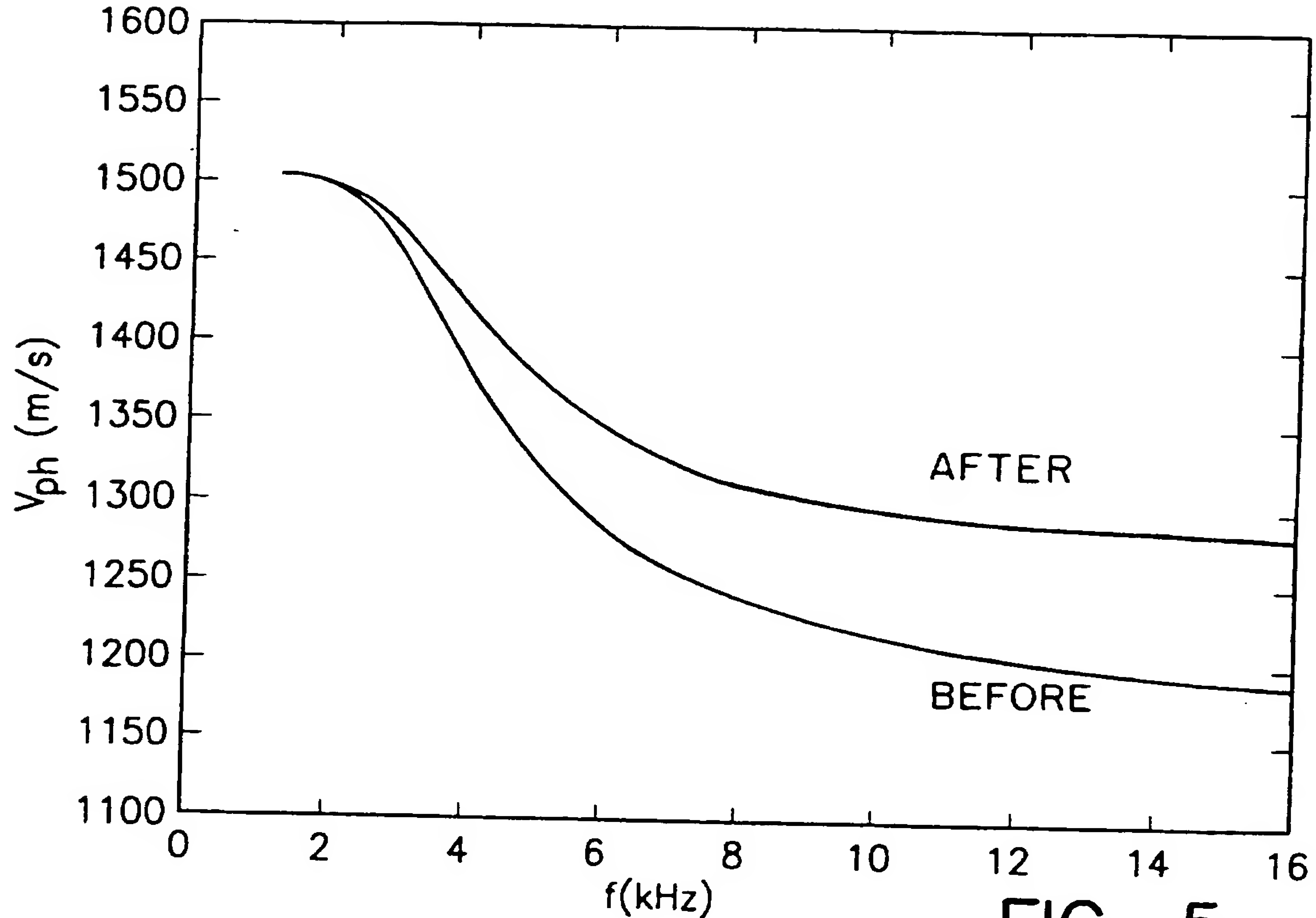
1600
1550
1500
1450
1400
1350
1300
1250
1200
1150
1100
$V_{ph}$ (m/s)
AFTER
BEFORE
0
2
4
6
8
10
12
14
16
f(kHz)

FIG. 5

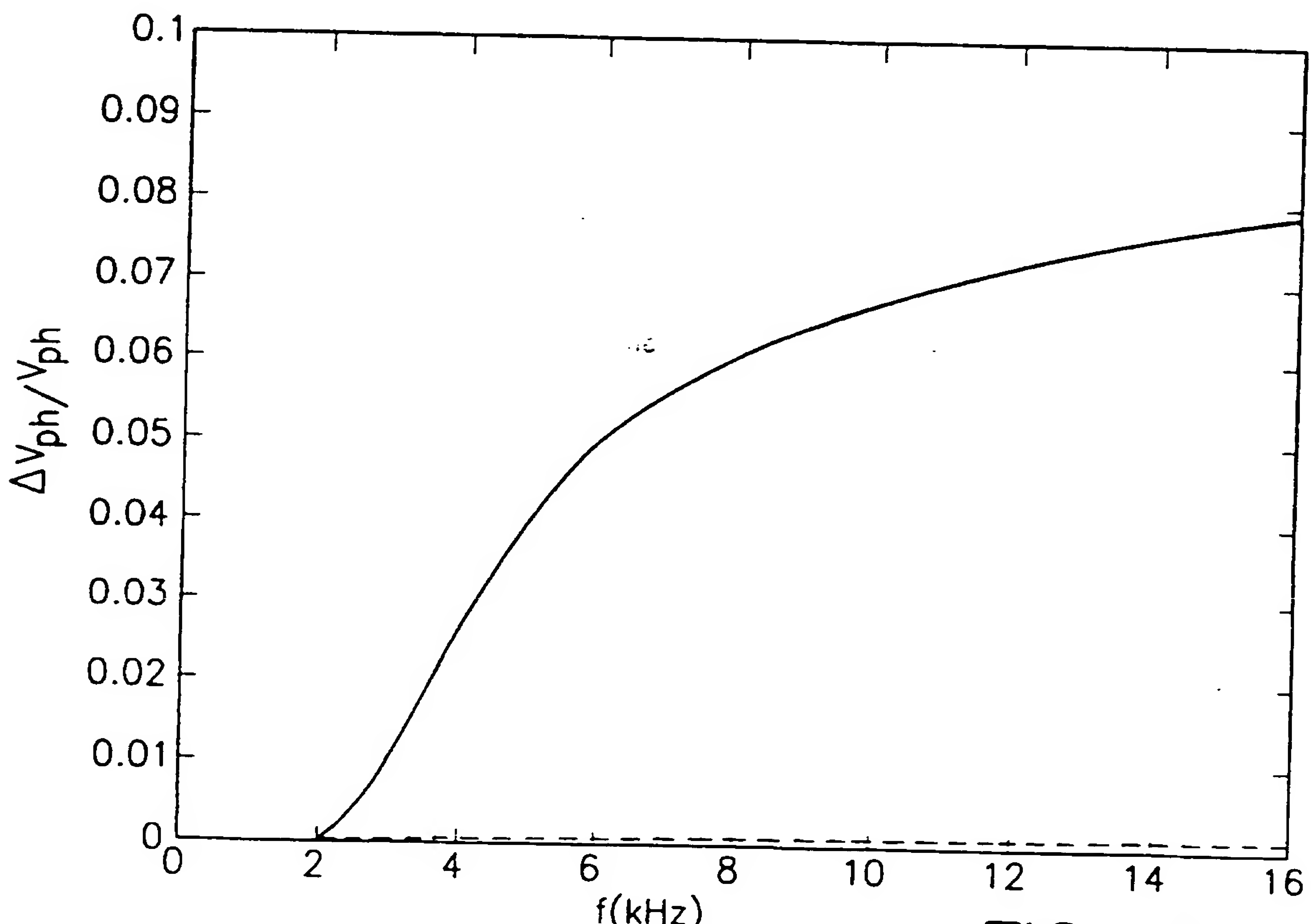
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
$\Delta V_{ph}/V_{ph}$
0
2
4
6
8
10
12
14
16
f(kHz)

FIG. 6

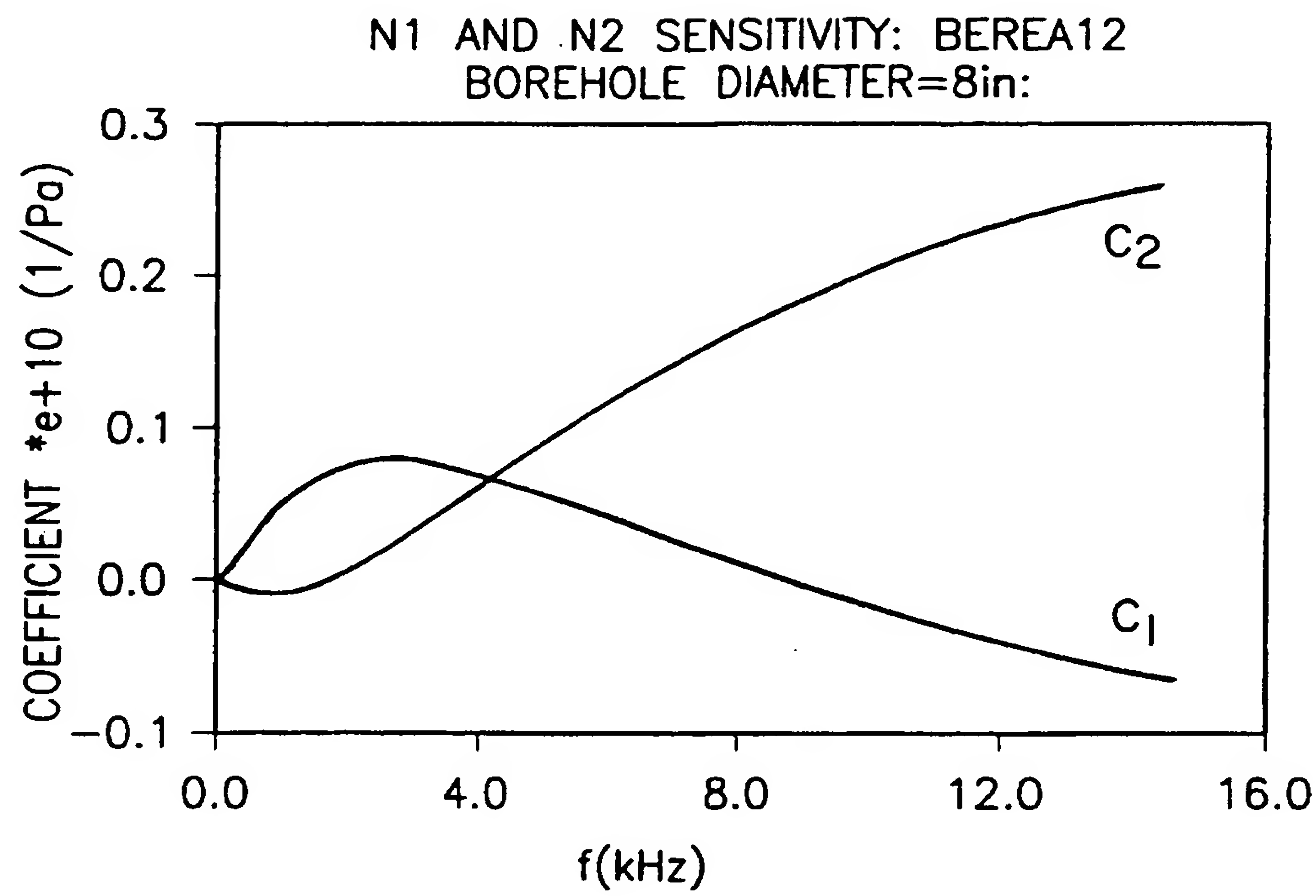
N1 AND N2 SENSITIVITY: BEREA12
BOREHOLE DIAMETER=8in:
COEFFICIENT *e+10 (1/Pa)
0.3
0.2
0.1
0.0
−0.1
0.0
4.0
8.0
12.0
16.0
f(kHz)
C2
C1

FIG. 7

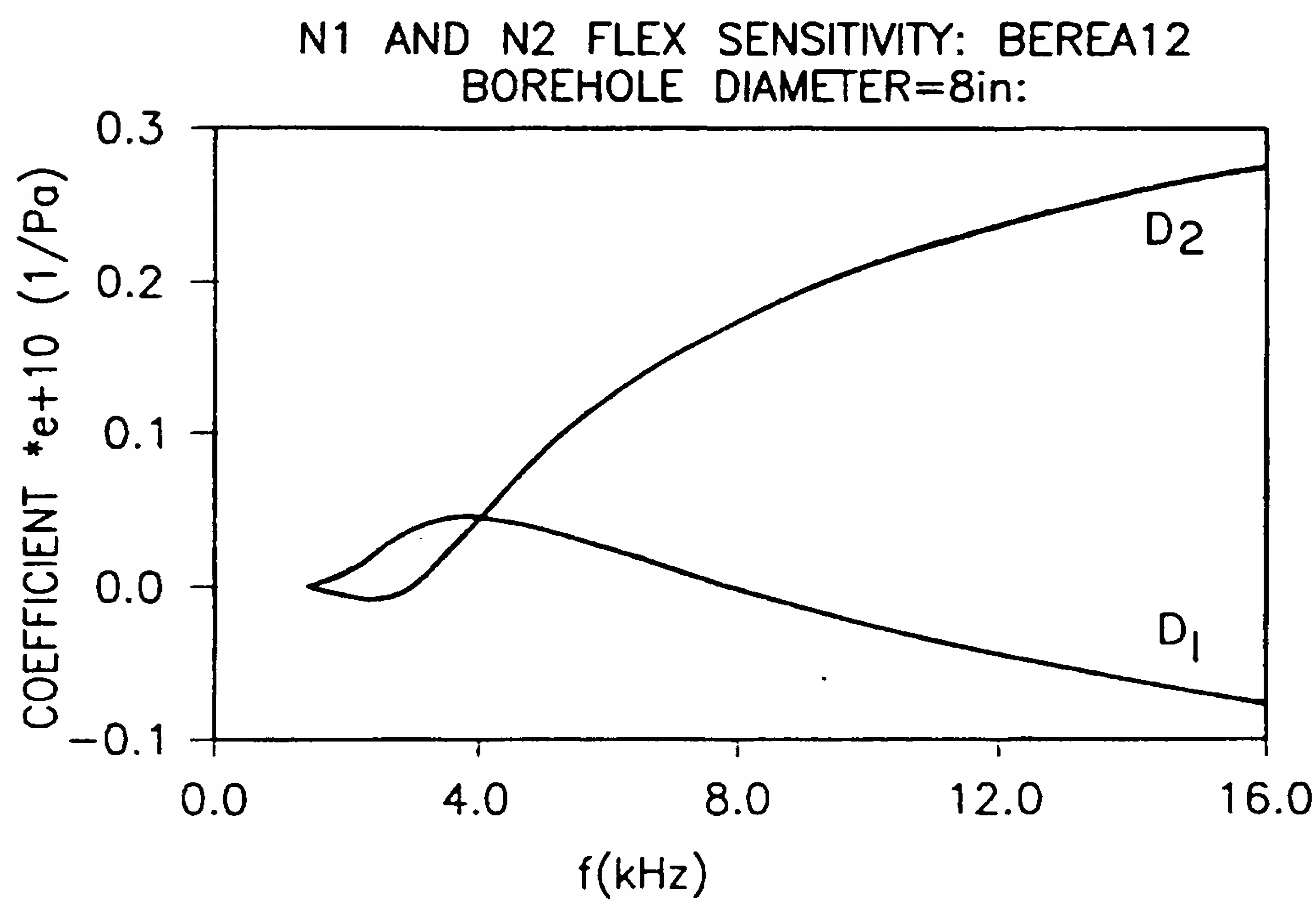
N1 AND N2 FLEX SENSITIVITY: BEREA12
BOREHOLE DIAMETER=8in:
COEFFICIENT *e+10 (1/Pa)
0.3
0.2
0.1
0.0
−0.1
0.0
4.0
8.0
12.0
16.0
f(kHz)
D2
D1

FIG. 8

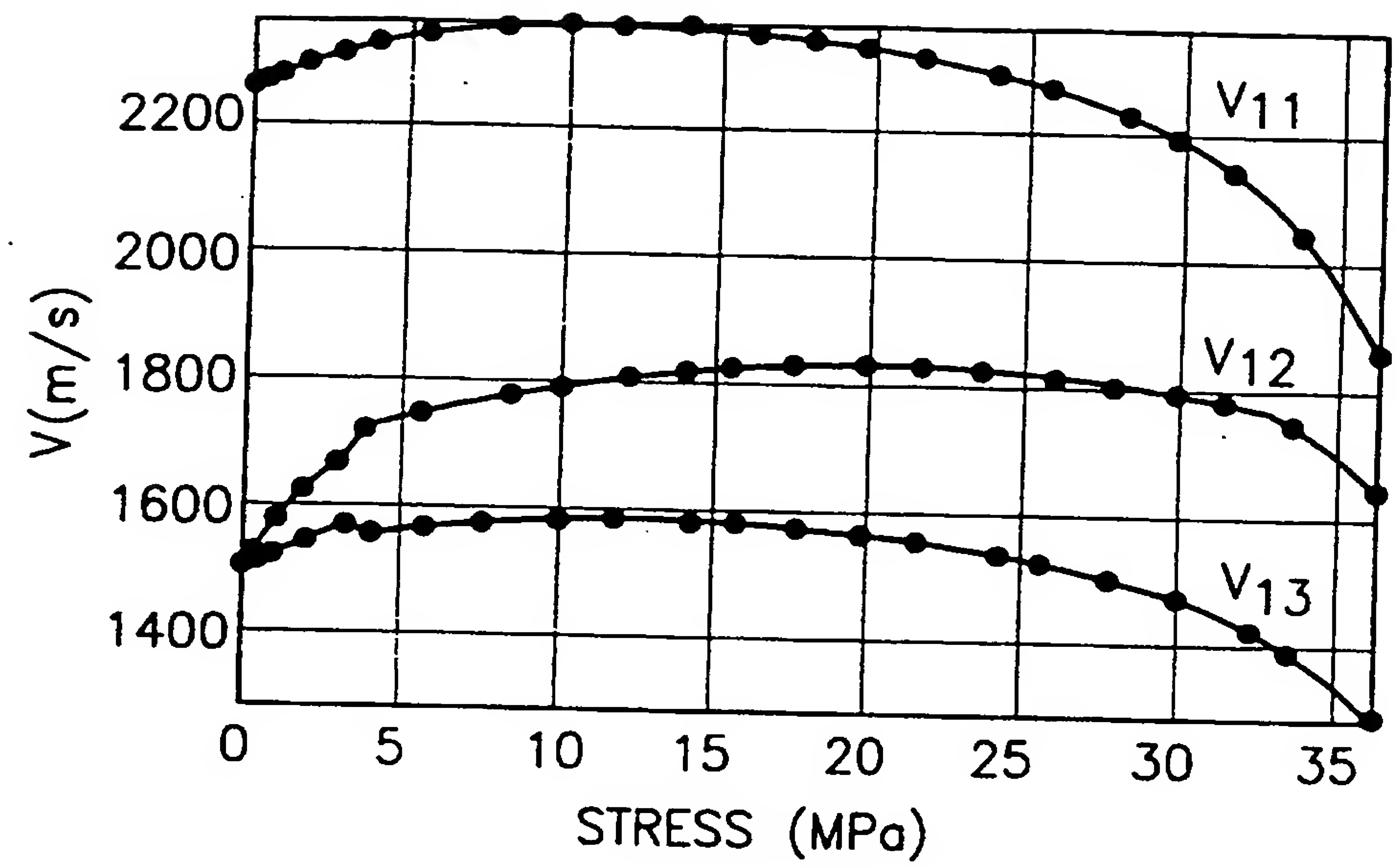
2200
2000
1800
1600
1400
V(m/s)
0
5
10
15
20
25
30
35
STRESS (MPa)
$V_{11}$
$V_{12}$
$V_{13}$

FIG. 9

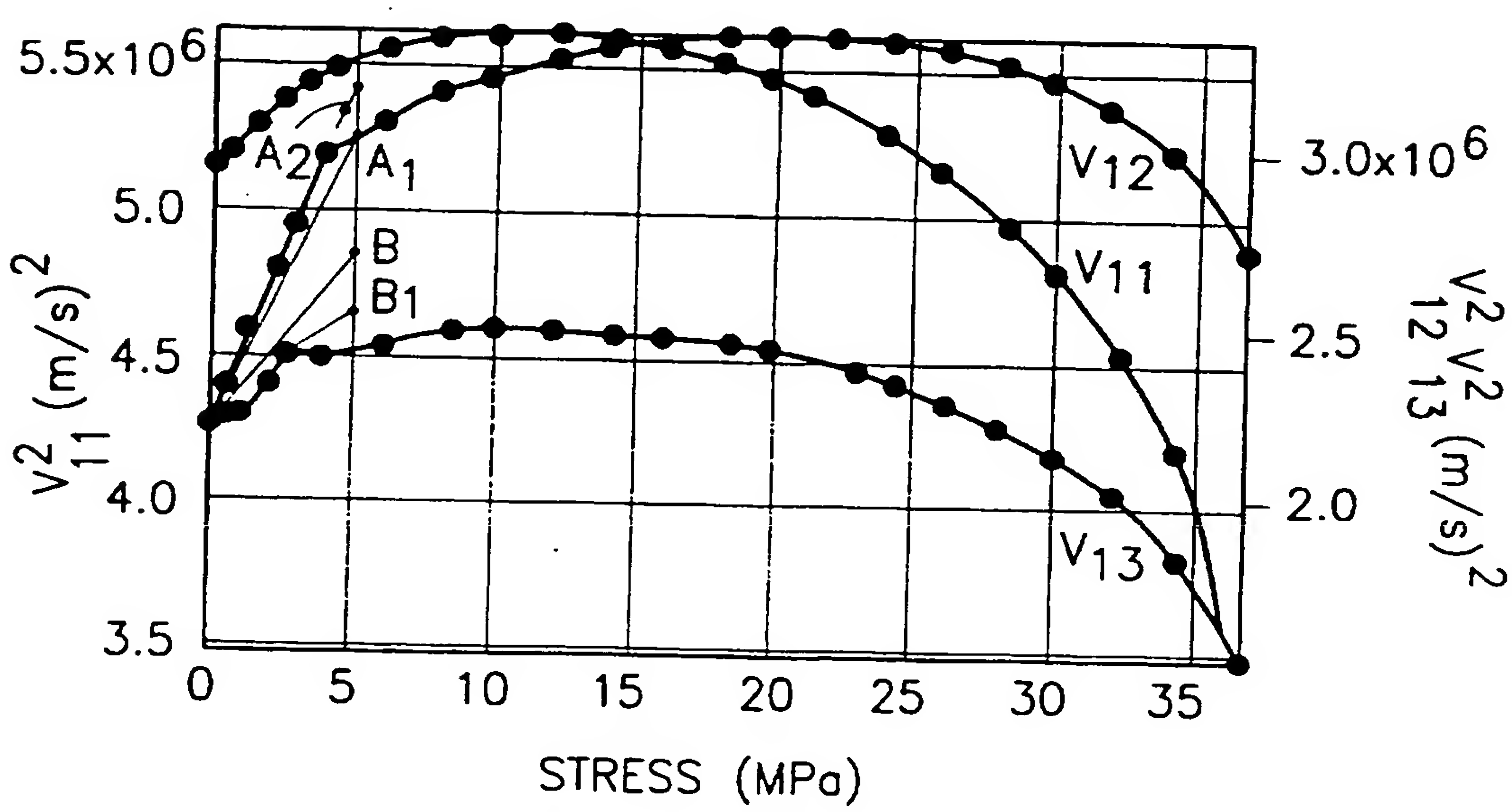
5.5x10^6
5.0
4.5
4.0
3.5
$V_{11}^2$ (m/s)$^2$
3.0x10^6
2.5
2.0
$V_{12}^2 V_{13}^2$ (m/s)$^2$
0
5
10
15
20
25
30
35
STRESS (MPa)
$A_2$
$A_1$
B
$B_1$
$V_{12}$
$V_{11}$
$V_{13}$

FIG. 10

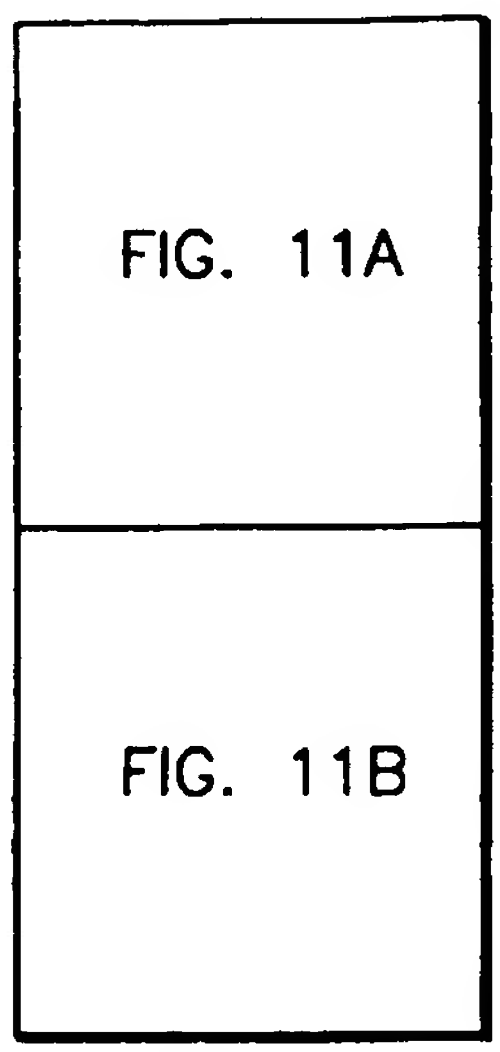


FIG. 11

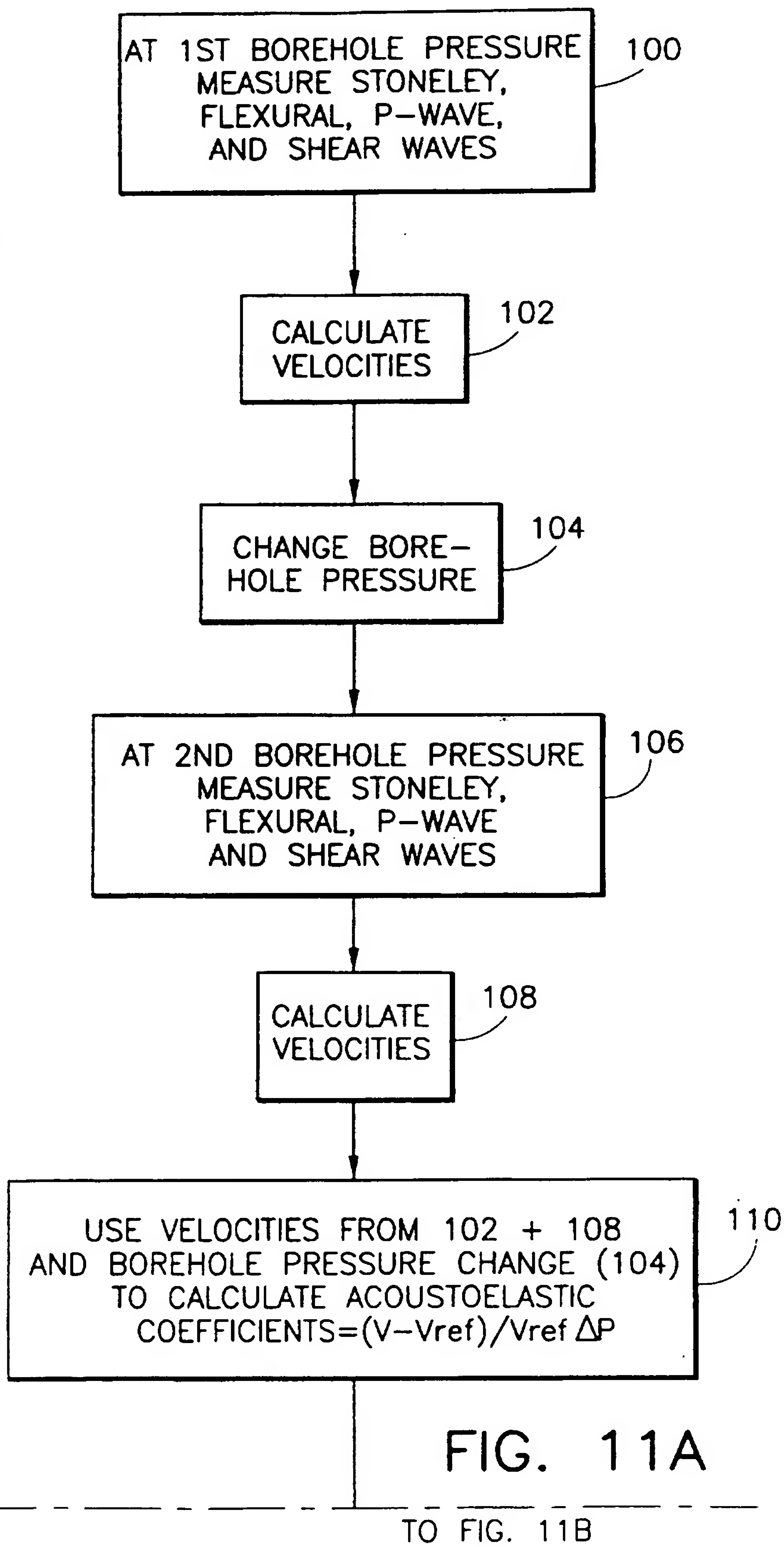


FIG. 11A

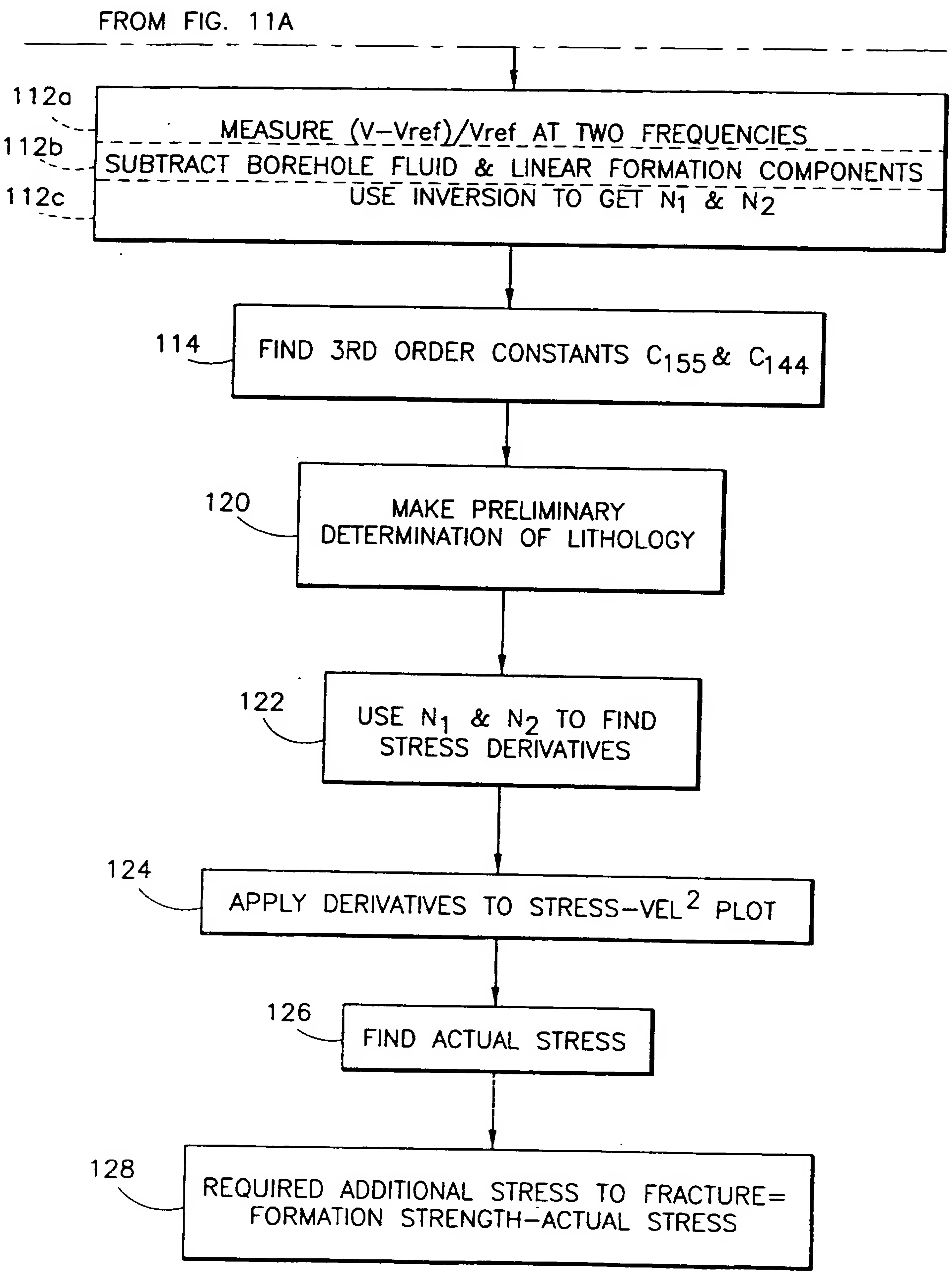
FROM FIG. 11A
112a
112b
112c
MEASURE (V−Vref)/Vref AT TWO FREQUENCIES
SUBTRACT BOREHOLE FLUID & LINEAR FORMATION COMPONENTS
USE INVERSION TO GET $N_1$ & $N_2$
114
FIND 3RD ORDER CONSTANTS $C_{155}$ & $C_{144}$
120
MAKE PRELIMINARY DETERMINATION OF LITHOLOGY
122
USE $N_1$ & $N_2$ TO FIND STRESS DERIVATIVES
124
APPLY DERIVATIVES TO STRESS−VEL$^2$ PLOT
126
FIND ACTUAL STRESS
128
REQUIRED ADDITIONAL STRESS TO FRACTURE= FORMATION STRENGTH−ACTUAL STRESS

FIG. 11B

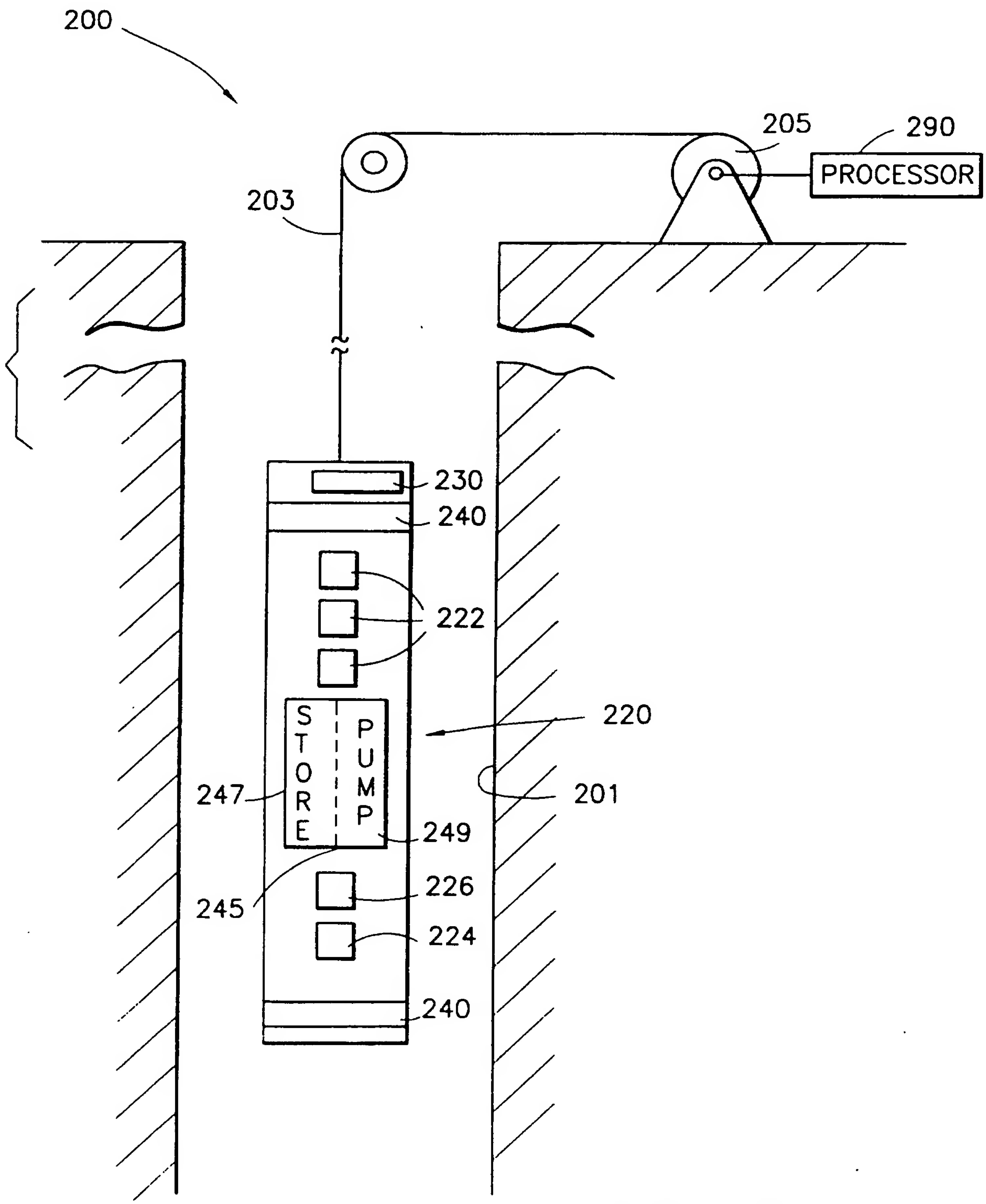
200
205
290
PROCESSOR
203
230
240
222
220
S T O R E
P U M P
247
201
249
245
226
224
240

FIG. 12

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# MEASUREMENT OF NONLINEAR PROPERTIES OF AN EARTH FORMATION

This invention relates broadly to methods and apparatus for investigating subsurface earth formations. More particularly, this invention relates to sonic borehole tools and methods for measuring a nonlinear parameter of an earth formation. The invention has particular application in using the nonlinear parameter measurements for determining in situ the strength of rocks, which information is useful in the production of oil from the formation, although the invention is not limited thereto and provides other useful information regarding formation parameters.

The art of sonic well logging for use in determining formation parameters is a well established art. Sonic well logs are typically derived from sonic tools suspended in a mud-filled borehole by a cable. The tools typically include a sonic source (transmitter) and a plurality of receivers which are spaced apart by several inches or feet. Typically, a sonic signal is transmitted from the transmitter at one longitudinal end of the tool and received by the receivers at the other, and measurements are made every few inches as the tool is drawn up the borehole. Depending upon the type of transmitter or source utilized (e.g. dipole, monopole), the sonic signal generated by the transmitter travels up the borehole and/or enters the formation adjacent the borehole, and the arrival times of one or more of the compressional (P-wave), shear (S-wave), Stoneley (tube wave), and flexural wave can be detected by the receivers. The receiver responses are typically processed in order to provide a time to depth conversion capability for seismic studies as well as for providing the determinations of formations parameters such as porosity.

A method of using a sonic tool in conjunction with the changing of the pressure in the borehole is also known. In the defensive publication H1156 of Siegfried, it is suggested that compressional and shear wave speeds, amplitudes, and phase shifts of sonic waveforms be measured in the borehole during multiple runs by a sonic logging tool, where the borehole pressure is different for each of the runs. According to Siegfried, differences in any of these acoustic properties resulting from a change in pressure provides an indication of the relative fracturing of the formation.

While measurements of the compressional and shear waves are useful in quantifying and characterizing various parameters of the formation, including fracturing, it will be appreciated that to date, there has been no successful mechanism for making in situ

determinations of nonlinear aspects of the formation. For purposes of this invention, it should be understood that the term "nonlinear" when used to describe a material relates to the fact that a plot of stress versus strain in a material will exhibit some nonlinear behavior. Phenomenologically, the strain energy function $U(\varepsilon)$ of an isotropic elastic solid can be written as:

$$U(\varepsilon) = f(\lambda,\mu)\varepsilon^2 + g(\alpha,\beta,\gamma)\varepsilon^3 \qquad (1)$$

where $\varepsilon$ is the strain, $\lambda$ and $\mu$ are the second order elastic Lamé constants, and $\alpha$, $\beta$, and $\gamma$ are the third order elastic constants. From equation (1), it will be appreciated that the stress $\sigma$ is defined by:

$$\sigma = \partial U/\partial\varepsilon = f(\lambda,\mu)\varepsilon + g(\alpha,\beta,\gamma)\varepsilon^2 \qquad (2)$$

where f and g denote general functions of quantities in parentheses. Based on equation (2) it is seen that the second order Lamé constants are linear constants, while the third order constants are nonlinear, and hence measure the nonlinearity of the material. The more nonlinear the stress versus strain plot is, the more nonlinear the material is said to be. Various manifestations of non-linearity include: the varying of the acoustic velocity in the material when the confining pressure changes; the varying of the acoustic velocity in the material when the amplitude of the acoustic wave changes; the interaction of two monochromatic acoustic beams having different frequencies to create third and fourth acoustic beams having the difference frequency and the additive frequency of the two incident beams; and evidence of frequencies being generated within the material which were not part of any input signal.

In the oil production industry, rock properties such as sanding, fracturing and borehole collapse can be considered to relate to the nonlinear properties of the formation. In each case, the strain in the rock catastrophically exceeds that which would be expected from a linear stress-strain relationship. As suggested in the parent applications hereto, since the less consolidated a formation is, the more nonlinear it is, a measurement of the nonlinearity of the formation can provide a measurement of the relative state of the consolidation of the formation. As suggested above, whether a layer of a formation is well or poorly consolidated, can broadly affect the producibility of the layer and formation, as well as the manner in which production is to be carried out.

It is therefore an object of the invention to provide a measurement of the nonlinearity of a formation traversed by a borehole.

According to the present invention, there is provided a method of providing indications of formation nonlinearities by using determinations of the acoustic velocity of the formation when the borehole traversing the formation is placed under different static pressures.

It is a further aspect of the invention to provide indications of formation nonlinearities by measuring the Stoneley, or flexural wave velocities both before and after a change in the static pressure of a borehole, and by subtracting out the nonlinear contribution of the borehole fluid from the determinations.

It is an additional aspect of the invention to determine values of acoustoelastic coefficients of a nonlinear formation at a given frequency which are related to the nonlinear constants of the formation by measuring Stoneley or flexural wave velocities both before and after a change in the static pressure of a borehole.

Another aspect of the invention is to determine values of nonlinear constants of a formation from values of acoustoelastic coefficients determined by measuring Stoneley and/or flexural wave velocities both before and after a change in the static pressure of a borehole.

A further aspect of the invention is to determine in situ a rock strength of a formation by utilizing nonlinear constants of the formation which were determined by measuring Stoneley and/or flexural wave velocities both before and after a change in the static pressure of a borehole.

An additional aspect of the invention is to utilize determined linear and nonlinear constants of the formation to characterize the formation.

Yet another aspect of the invention is to use determined linear and nonlinear constants which characterize the formation to analyze the amount of additional stress which would be required to fracture the formation.

The method of the invention broadly comprises measuring at a plurality of different static borehole pressures the acoustic Stoneley and/or flexural wave velocities of waves propagating through the borehole and formation, and generating an indication of the nonlinearity of the formation by processing the velocity measurements. The processing of the velocity measurements includes either determining a fractional change in the measured

acoustic velocity and dividing the change by the change in the borehole pressure to obtain frequency dependent acoustoelastic coefficients which are an indication of nonlinearity, or determining the fractional change in the measured acoustic velocity, and subtracting from the fractional change a component generated by the borehole fluid and a component due to linear aspects of the formation to provide a nonlinear formation component. The nonlinear formation components obtained at a plurality of frequencies are used to find nonlinear parameters of the formation. In addition, the nonlinear parameters may then be used in conjunction with shear wave speed information and a database of experimental data, to determine the stress in the formation, the strength of the formation, and therefrom, the amount of additional stress required to fracture the formation.

The system of the invention relates directly to the preferred methods of the invention. The system of the invention preferably includes a borehole tool, a borehole pressurizing means, and a processing means. The borehole tool includes a monopole and/or dipole source which produce dispersive Stoneley and/or flexural waves in the borehole and one or more monopole or dipole detectors (e.g. hydrophones) which are spaced from the acoustic source and which measure the magnitude of the resulting waves, and means for processing and/or sending the measurements uphole, typically via a wireline.

The borehole pressurizing means preferably comprises a packer type device in conjunction with fluid injection means located on the borehole tool for pressurizing a portion of the borehole, or a packer type device located on a well head which pressurizes the entire borehole.

The processing means of the invention is typically a VAX computer manufactured by Digital Equipment Corporation of Brainard, Massachusetts, or the like coupled to the wireline, which receives the measurements provided from the downhole means for processing and/or sending and processes the information. In particular, the processing means preferably utilizes Prony's method to determine the velocity of the received waves as a function of frequency, and then processes that information to determine frequency dependent acoustoelastic coefficients or frequency independent nonlinear parameters of the formation. It should be noted that both the acoustoelastic coefficients and nonlinear parameters of the formation are functions of in situ stresses in the reference state of the formation. The acoustoelastic coefficients are obtained by determining the fractional change in the measured acoustic velocities, and dividing the fractional change by the change in static borehole pressure. The nonlinear parameters are obtained by determining at a plurality

of frequencies the fractional change in the measured acoustic velocities, and subtracting components generated by the borehole fluid and by the linear aspects of the formation from the fractional changes in order to provide nonlinear formation components. The nonlinear formation components are then processed to find nonlinear parameters ($N_1$ and $N_2$) of the formation. In turn, the nonlinear parameters may be processed in conjunction with shear wave data and a database of experimental data, to determine the stress in the formation, the strength of the formation, and therefrom, the amount of additional stress required to fracture the formation.

According to one preferred aspect of the invention, the nonlinear parameters of the formation are determined according to an equation:

$$\left(v^{Stoneley} - v_{ref}^{Stoneley}\right) / v_{ref}^{Stoneley} = C_1 N_1 + C_2 N_2 + \Delta v/v\big|_{fluid} + \Delta v/v\big|_{linear} \quad (3)$$

where $\left(v^{Stoneley} - v_{ref}^{Stoneley}\right) / v_{ref}^{Stoneley} \Delta P$ is the frequency dependent acoustoelastic coefficient, $v_{ref}^{Stoneley}$ is the velocity of the Stoneley (or flexural) wave in an unpressurized borehole (or in a borehole at a given reference pressure), $v^{Stoneley}$ is the measured velocity of the dispersive Stoneley (or flexural) wave at a known pressure, $\Delta P$ is the difference in pressure between the reference pressure and the known pressure, $\Delta v/v\big|_{linear}$ is the portion of the fractional change in the Stoneley (or flexural) dispersion caused by an increase in the borehole pressure that can be calculated from the linear constants of the formation in the ambient state, $\Delta v/v\big|_{fluid}$ is the portion of the fractional change in the Stoneley (or flexural) dispersion caused by an increase in the borehole pressure that can be calculated from the known borehole fluid nonlinearity in the ambient state, $C_1$ and $C_2$, are volume integrals which are a function of frequency and are calculable in terms of the Stoneley (or flexural) wave solution in the ambient state, and $N_1$ and $N_2$, are nonlinear parameters of the formation.

According to another preferred aspect of the invention, frequencies in the range of 3 kHz to 6 kHz are utilized in determining the nonlinear parameters. Further, utilizing the nonlinear parameters in conjunction with the change in pressure, a multi-frequency inversion of the Stoneley or flexural wave velocity dispersions according to $AX = B$ yields a determination of two normalized nonlinear constants ($c_{144}$ and $c_{155}$) of the formation, where

$$A = \begin{vmatrix} c_1^{f1} c_2^{f1} \\ c_1^{f2} c_2^{f2} \end{vmatrix} \qquad (4)$$

$$X = \begin{vmatrix} N_1 \Delta P \\ N_2 \Delta P \end{vmatrix} \qquad (5)$$

$$B = \begin{vmatrix} (\Delta v/v|_{Stoneley} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f1} \\ (\Delta v/v|_{Stoneley} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f2} \end{vmatrix} \qquad (6)$$

and where $N_1 = -c_{144}/c_{66}$ and $N_2 = -c_{155}/c_{66}$, where $c_{66}$ is the formation shear modulus.

According to yet another preferred aspect of the invention, the stress derivatives of the square of the shear velocities in the formation as a function of stress can be determined from the determined nonlinear parameters $N_1$ and $N_2$ (or the related nonlinear constants $c_{144}$ and $c_{155}$) and from the determinable linear constants of the formation according to:

$$\frac{\rho_0 \partial V_{12}^2}{\partial S} = \frac{(2 - N_2)c_{66}}{Y} + \frac{(N_1 + N_2)\upsilon c_{66}}{Y} \qquad (7)$$

$$\frac{\rho_0 \partial V_{13}^2}{\partial S} = \frac{(\upsilon N_2 - N_2)c_{66}}{Y} + \frac{(N_2 - 2)\upsilon c_{66}}{Y} \qquad (8)$$

where $\rho_0$ is the formation mass density, $V_{12}$ and $V_{13}$ are the two shear wave velocities for propagation along direction a first direction ("1") and polarization along perpendicular directions thereto ("2" and "3" respectively), $S$ is the uniaxial stress along the one of the perpendicular directions, $Y$ is Young's modulus (i.e. a linear parameter of the formation given by $Y = c_{66}(3c_{12} + c_{66})/(c_{12} + c_{66})$, and $\upsilon$ is Poisson's ratio (i.e. another linear parameter of the formation) given by $\upsilon = c_{12}/(2c_{12} + c_{66})$. The linear parameters are determined from the compressional and shear wave velocities and from the formation mass density. When the slope of the stress derivatives of $\rho_0 V_{12}^2$, and $\rho_0 V_{13}^2$ are positive, the stress in the formation is considered to be well below the formation strength, while when the slope of the stress derivatives are negative, the existing stress in the formation is considered to be near the formation strength. From experimental data regarding the compressional and shear wave velocities of rock formations as a function of uniaxial stress and the shear strength, and by utilizing the stress derivatives as calculated, the rock stress is obtained.

From the stress in the rock, and from the experimental data regarding the strength of the rock, a determination can be made regarding the amount of additional stress which will cause the rock to fail (i.e., fracture).

Additional objects and advantages of the invention will become apparent to those skilled in the art upon reference to the detailed description taken in conjunction with the provided figures, in which:

Figure 1 is a plot of dispersion curve as a function of frequency and velocity for a Stoneley wave in a first formation before and after pressurization;

Figure 2 is a plot of a dispersion curve as a function of frequency and velocity change for a Stoneley wave in a first formation before and after pressurization;

Figure 3 is a plot of dispersion curve as a function of frequency and velocity for a Stoneley wave in a second formation before and after pressurization;

Figure 4 is a plot of a dispersion curve as a function of frequency and velocity change for a Stoneley wave in a second formation before and after pressurization;

Figure 5 is a plot of a dispersion curve as a function of frequency and velocity for a flexural wave in the second formation before and after pressurization;

Figure 6 is a plot of a dispersion curve as a function of frequency and velocity change for a flexural wave in the second formation before and after pressurization;

Figure 7 is a plot showing the frequency sensitivity of coefficients $C_1$ and $C_2$ to fractional changes in the Stoneley wave velocity caused by a borehole pressure increase of unit magnitude in the second formation;

Figure 8 is a plot showing the frequency sensitivity of coefficients $D_1$ and $D_2$ to fractional changes in the flexural wave velocity caused by a borehole pressure increase of unit magnitude in the second formation;

Figure 9 is a plot of the plane wave velocities as a function of stress in the second formation;

Figure 10 is a plot of the square of the plane wave velocities as a function of stress in the second formation;

Figure 11 is a block diagram of the preferred method of the invention for measuring nonlinear properties of an earth formation; and

Figure 12 is a schematic diagram of the system of the invention which carries out the method according to Fig. 11.

Before describing the method and system of the invention in detail, an understanding of the theoretical underpinnings of the invention is helpful. In reviewing the theoretical

underpinnings, however, various aspects of the invention should be kept in mind. In particular, according to the objects of the invention, it is desired to use measurements made of the acoustic Stoneley wave or flexural wave velocities in a borehole traversing a formation both before and after a change in the static pressure of a borehole in order to determine one or more of: acoustoelastic coefficients of the formation; nonlinear parameters of the formation; the stress in the formation; and the amount of additional stress which would be required to fracture the formation. The propagation of Stoneley and flexural waves in a fluid-filled borehole in an ambient condition is adequately described by the linear equations of elasticity. However, when the borehole fluid is pressurized, such as with the aid of packers at the wellhead, both the fluid and the surrounding formation are subjected to biasing stresses. These biasing stresses in the propagating medium result in corresponding changes in the effective elastic stiffnesses, mass density and path length between two observation (detection) points. Under this situation, the Stoneley and flexural wave speeds will change as a function of the increase in the borehole pressure and the degree of nonlinearity of the surrounding formation and borehole fluid. The dependence of the acoustic wave velocity on biasing stresses in the propagating medium is referred to as the acoustoelastic phenomenon, (see Hughes, D.S. and J.L. Kelley, "Second-order Elastic Deformation of Solids", Physics Review 92 pp 1145-1149 (1953); Toupin, R.A., and B. Bernstein, "Sound Waves in Deformed Perfectly Elastic Materials - Acoustoelastic Effect", J. Acoust. Soc. Am. 33, pp. 216-225 (1961); Thurston, R.N. and K. Brugger, "Third-Order Elastic Constants and the Velocity of Small Amplitude Elastic Waves in Homogeneously Stressed Media", Physics Review 133, pp. A1604-1610 (1964)), and characteristics of the formation may be described according to acoustoelastic coefficients of the formation.

The acoustoelastic phenomenon may be described by first considering that the propagation of a small amplitude wave in an isotropic and homogeneous medium is governed by the linear equations of motion. When the propagating medium is prestressed, the propagation of such waves are properly described by equations of motion for small dynamic fields superimposed on a static bias. A static bias represents any statically deformed state of the medium due to an externally applied load or residual stress. The equations of motion for small dynamic fields superimposed on a static bias are derived from the rotationally invariant equations of nonlinear elasticity by making a Taylor expansion of the quantities for the dynamic state about their values in the biasing state.

When the biasing state is inhomogeneous, the effective elastic constants are position dependent and a direct solution of the boundary value problem is not possible. Under this situation, a perturbation procedure can readily treat spatially varying biasing states such as those due to a nonuniform radial stress distribution away from the borehole, and the corresponding changes in the Stoneley and flexural wave velocities can be calculated as a function of frequency. In particular, a modified Piola-Korchhoff stress tensor $P_{\alpha j}$ in a perturbation model yields the following expression for the first-order perturbation in the eigen frequency $w_m$ for a given wavenumber $k_z$:

$$\Delta\omega = \frac{\int_V \Delta P_{\alpha j} \upsilon^m_{j.\alpha} dV - \omega_m^2 \int_V \Delta\rho \upsilon^m_j \upsilon^m_j dV}{2\omega_m \int_V \rho_0 \upsilon^m_j \upsilon^m_j dV} \tag{9}$$

where
$$\Delta P_{\alpha j} = H_{\alpha j\gamma\beta} \upsilon^m_{\gamma.\beta} \tag{10}$$

$$H_{\alpha j\gamma\beta} = h_{\alpha j\gamma\beta} + T_{\alpha\beta}\delta_{j\gamma} + \Delta P\left(\delta_{\alpha j}\delta_{\gamma\beta} - \delta_{\alpha\gamma}\delta_{j\beta}\right) \tag{11}$$

$$\begin{aligned} h_{\alpha j\gamma\beta} = {} & -c_{\alpha j\gamma\beta} w_{\delta.\delta} + c_{\alpha j\gamma\beta AB} E_{AB} \\ & + w_{\alpha.L} c_{Lj\gamma\beta} + w_{j.M} c_{\alpha M\gamma\beta} \\ & + w_{\gamma.P} c_{\alpha jP\beta} + w_{\beta.Q} c_{\alpha j\gamma Q} \end{aligned} \tag{12}$$

$$T_{\alpha\beta} = c_{\alpha\beta\gamma\delta} w_{\delta.\gamma} \tag{13}$$

$$E_{AB} = \frac{1}{2}\left(w_{A.B} + w_{B.A}\right) \tag{14}$$

and where the Cartesian tensor notation, the convention that a comma followed by an index $P$ denotes differentiation with respect to $Xp$, and the summation convention for repeated tensor indices is used. Although $h_{\alpha j\gamma\beta}$ exhibits the usual symmetries of the second-order constants of linear elastlcity, the effective elastic stiffness tensor $H_{\alpha j\gamma\beta}$ does not have these properties as is evident from equation (11).

Before discussing the various quantities in equations (9)-(14), it should be appreciated that the present position of points of material (e.g. formation) may be written as

$$\underline{y}(\underline{X},t)=\underline{X}+\underline{w}(\underline{X})+\underline{u}(\underline{X},t) \tag{15}$$

where $\underline{w}$ denotes the displacement due to the applied static loading of material points with position vector $\underline{X}$ in the assumed ambient isotropic state (also called the reference state), and $\underline{u}$ denotes the dynamic displacement vector of material points above and beyond that due to the static deformation. The small field Piola-Korchhoff stress $P_{\alpha i}$ in the intermediate state can be decomposed into two parts:

$$P_{\alpha j} = P^{L}_{\alpha j} + \Delta P_{\alpha j} \tag{16}$$

where $\Delta P_{\alpha j}$ is defined by equations (10) - (14), the superscript $L$ denotes the linear component of the stress tensor, and the linear component is defined according to:

$$P^{L}_{\alpha j} = c_{\alpha j \gamma \beta} u^{m}_{\beta,\gamma} \tag{17}$$

In equations (12), (13), and (17), the quantities $c_{\alpha j\gamma\beta}$ and $c_{\alpha j\gamma\beta AB}$ are respectively the second and third order elastic constants of the material, with Brugger's notation being utilized for the nonlinear constants. Brugger, K. "Thermodynamic Definitions of Higher-Order Elastic Coefficients", Physical Review 133, No. 6A pp. A1611-A1612. The various relationships among the various notations for the nonlinear constants that appear in the literature are known. Kostek, S. et al., "ThirdOrder Elastic Constants for an Inviscid Fluid", J. Acoust. Soc. Am. Vol 94 pp. 3014-3017 (Nov. 1993). In equations (11) - (14), $T_{\alpha\beta}$, $E_{AB}$, and $w_{\delta,\gamma}$ denote the biasing stresses, strains, and static displacement gradients respectively. The quantity $\Delta P$ denotes the increase in the borehole pressure.

In equation (9), $\Delta P_{\alpha j}$ are the perturbations in the Piola- Korchhoff stress tensor elements from the linear portion $P^{L}_{\alpha j}$ for the reference isotropic medium before the application of any biasing stresses; $\rho_0$ is the mass density of the propagating medium in the reference state; and $u^{m}_{j}$ represents the eigen-solution for the reference isotropic medium for a selected propagating mode. The index "$m$" refers to a family of normal modes for a borehole in an isotropic formation. The frequency perturbations $\omega$ are added to the elgen-frequency $\omega_m$ for various values of the wavenumber along the borehole axis ($k_z$) to obtain the final dispersion curves for the biased state. Thus, the modal solution is obtained. Although this method of solution is valid for all modes, of particular interest are the Stoneley ($m=0$), and flexural ($m=1$) modes.

An important feature of the perturbation equation (9) is that the volume integral in the numerator can be separated into two independent contributions coming from the borehole fluid and the formation. The advantage of this feature is that the borehole fluid induced velocity change, if known or determined, may be subtracted from the total velocity change in order to obtain the acoustoelastic response of the formation.

Using the numerical theory set forth above, computational results have been obtained for the Stoneley and flexural wave dispersion curves before and after pressuri2ation of a fluidfilled borehole surrounded by an isotropic nonlinear formation. Two distinct formations with material properties as listed in Table I below were considered as having a borehole of eight inch diameter, with formation I being a "fast" formation, and formation II being a moderately "slow" formation:

TABLE I

| Formation | $c_{11}$ GPa | $c_{66}$ Gpa | $\rho$ kg/m$^3$ | $v_p$ m/s | $v_s$ m/s | $c_{111}$ GPa | $c_{112}$ GPa | $c_{123}$ GPa | $\beta$ |
|---|---|---|---|---|---|---|---|---|---|
| I | 19.5 | 6.50 | 2135 | 3022 | 1745 | -3467 | -1155 | -1541 | -87 |
| II | 11.1 | 4.64 | 2062 | 2320 | 1500 | -21217 | -3044 | 2361 | -954 |

The nonlinearity parameter defined as $\beta=(3c_{11}+c_{111})/2c_{11}$ is substantially different for the two formations. This nonlinearity parameter for rocks varies by several orders of magnitude depending on the rock type, porosity, etc., and is substantially larger in granular materials as opposed to non- granular materials.

The borehole fluid is assumed to have a compressional wave speed of 1500 m/s and a mass density of 1000 kg/m$^3$. The nonlinear behavior of the fluid is usually expressed in terms of the parameters A and B appearing in the equation of state, where A is the bulk modulus of the fluid. The ratio of parameters B/A for borehole fluid can be taken to be set according to B/A = 5, and the corresponding third-order elastic constants are given by $c_{111}$= -22.5 GPa, $c_{112}$ = -13.5 GPa, and $c_{123}$ = -9.0 GPa.

When the borehole pressure is increased by $\Delta P$, static deformations of the borehole fluid and formation are governed by the static equations of equilibrium and continuity of radial component of particle displace formation yields the following biasing displacements, stresses, and strains:

$$w_R = \frac{\Delta P a^2}{2c_{66}R}, w_Z = 0 \quad (18)$$

$$T_{RR} = -\frac{\Delta P a^2}{R^2}, T_{\theta\theta} = \frac{\Delta P a^2}{R^2}, T_{ZZ} = 0 \quad (19)$$

$$E_{RR} = -\frac{\Delta P a^2}{2c_{66}R^2}, E_{\theta\theta} = \frac{\Delta P a^2}{2c_{66}R^2}, E_{ZZ} = 0 \quad (20)$$

where $a$ is the borehole radius, $R$ is the radial distance from the borehole axis, and $c_{66}$ is the formation shear modulus. Since the stresses and strains in the formation decrease according to the square of the radius, formation stresses will have less influence on the Stoneley and flexural waves at low frequencies which investigate deeper into the formation; i.e. high frequency Stoneley and flexural waves which do not penetrate as far into the formation will be more affected by formation stresses which exist due to the borehole.

It should be appreciated that the volume integrals of equation (9) extend over both the borehole fluid and the solid formation. However, the fluid contribution to the first-order perturbation in the eigen-frequency $\omega_m$ can be readily computed from the expression:

$$\Delta\omega_{fluid} = \frac{\int\limits_{V_f} \Delta c^f_{\alpha\beta\gamma\delta} u^m_{\delta,\gamma} u^m_{\beta,\alpha} dV - \omega^2_m \int\limits_{V_f} \Delta\rho^f u^m_\gamma u^m_\gamma dV}{2\omega \int\limits_{V_f} \rho^f u^m_j u^m_j dV} \qquad (21)$$

where

$$\Delta c^f_{\alpha\beta\gamma\delta} = \frac{\partial A}{\partial P}\Delta P \delta_{\alpha\beta}\delta_{\gamma\delta} = \left(1 + \frac{B}{A}\right)\Delta P \delta_{\alpha\beta}\delta_{\gamma\delta} \qquad (22)$$

$$\Delta\rho^f = \frac{\rho^f}{A}\Delta P \qquad (23)$$

and $\rho^f$ is the mass density of the fluid in the reference state, $B$ is the fluid nonlinear parameter appearing in the equation of state (Kostek, S. et al., "Third-order Elastic Constants for an Inviscid Fluid", J. Acoust. Soc. Am. Vol 94, pp. 3014-3017 (Nov. 1993)), and $V_f$ denotes the fluid volume. The fractional change in phase velocity resulting from changes in the effective stiffnesses and mass density of the material takes the form:

$$\Delta v / v = \Delta\omega / \omega_m \qquad (24)$$

for a fixed wavenumber.

Using equation (9) which includes contributions from both the borehole fluid and the solid formation. Stoneley wave dispersion curves in formation I are shown in Figure 1 both before and after borehole pressurization ($\Delta P$ = 13.79 MPa = 2000 psi). The dashed line in Fig. 1 denotes the contribution of the borehole fluid to the total velocity change-as obtained from equation (21). It should be appreciated that in the low frequency limit, the change in the tube wave speed after pressurization is essentially due to the acoustoelastic effect of the borehole fluid. Similarly, in Fig. 2, the fractional change in the Stoneley wave phase velocity as a function of frequency is seen for formation I. As expected, $\Delta v / v$ increases

with frequency because the acoustic field becomes more confined to the borehole wall. However, the acoustoelastic contribution from the borehole fluid decreases with increasing frequency.

The Stoneley wave dispersion curves and fractional change in the Stoneley wave phase velocity for formation II are seen in Figures 3 and 4 respectively. The rock parameters for this formation correspond to those of a dry Berea sandstone. Since formation II exhibits a higher nonlinearity than formation I, even for a relatively small pressure difference ($\Delta P$ = 3.4 MPa =500 psi), significant changes are seen in the Stoneley wave speed. The sharp increase in the Stoneley wave speed due to a pressure increase at all frequencies for formation II results from the high degree of rock nonlinearity, and it will be appreciated that the fluid contribution is small compared to the formation contribution. It will also be appreciated that the fractional change in the Stoneley wave speed in formation II is significantly higher than in formation I, even when the pressure difference is considerably smaller. Thus, in formation II, at 4kHz, the Stoneley wave speed of the formation increases approximately four percent when the pressure is increased 500 psi .

The flexural wave dispersion curves and fractional change in the flexural wave phase velocity for formation II are seen in Figures 5 and 6 respectively. As is seen in both Figures 5 and 6, the fluid contribution is negligibly small due to the relatively low nonlinearity of the fluid when compared to the formation.

The sensitivity of the two formation nonlinear constants $N_1$ and $N_2$ to the Stoneley dispersion caused by an increase in the borehole pressure is obtained from equation (9). Substituting the linear (second-order) as well as nonlinear (third-order) elastic constants for an isotropic medium subject to the biasing stresses caused by an increase in the borehole pressure together with the Stoneley wave solution $u^m$ ($m$=0) into equation (9), the change of phase velocity at a given frequency can be expressed as set forth above in equation (3):

$$\left(v^{Stoneley} - v^{Stoneley}_{ref}\right) / v^{Stoneley}_{ref} = C_1 N_1 + C_2 N_2 + \Delta v/v\big|_{fluid} + \Delta v/v\big|_{linear} \quad (25)$$

where $$C_1 = \frac{1}{2\omega_m^2 J_N}\left(J_1 - \frac{J_3}{2}\right) \quad (26)$$

$$C_2 = \frac{1}{2\omega_m^2 J_N}\left(2J_1 + \frac{J_3}{2}\right) \quad (27)$$

$$N_1 = \frac{c_{123} - c_{112}}{2c_{66}} \tag{28}$$

$$N_2 = \frac{c_{112} - c_{111}}{4c_{66}} \tag{29}$$

and where compressed Voigt's notation has been used for both the linear and nonlinear constants. Since there are only three independent nonlinear (third-order) elastic constants for an isotropic material, other third-order elastic constants, such as $c_{144}$ and $c_{155}$ can be also expressed in terms of $c_{111}$, $c_{112}$, and $c_{123}$ such as:

$$c_{155} = (1/4)(c_{111} - c_{112}) \tag{30}$$

$$c_{144} = (1/2)(c_{112} - c_{123}) \tag{31}$$

It can be seen from equations (28) - (31) that $N_1 = -c_{144}/c_{66}$ ,, and $N_2 = -c_{155}/c_{66}$. It can also be seen from equations (26) and (27) that $C_1$ and $C_2$ are frequency dependent, and their frequency dependence for a unit increase in borehole pressure (i.e., $\Delta P = -1 N/m^2$) is shown in graphical form in Fig. 7.

It is clear from Fig. 7 that the magnitude of $C_2$, is approximately zero over a frequency band of 0 to 2 kHz. Therefore, inversion for the nonlinear constant $N_2$ of the formation from the fractional changes in the Stoneley wave velocities in this frequency band would result in large errors in its estimated values. Thus, a preferred frequency band for the inversion of $N_1$ and $N_2$ is one where both the values of $C_1$ and $C_2$ are significantly large. However, it is known in the art that a conventional sonic tool with sources and receivers placed on the borehole axis is not very efficient in recording Stoneley wave signals at frequencies much higher than about 10 kHz (see, Kurkjian, A.L., "Numerical Computation of Individual Far-Field Arrivals Excited by an Acoustic Source in a Borehole", Geophysics Vol. 50, No. 5, pp. 852-866 (May 1985)). Using this information as well as the sensitivity analysis of Fig. 7, an optimal frequency band for processing the Stoneley wave velocity changes caused by a borehole pressurization for the determination of $N_1$ and $N_2$ will preferably be between approximately 2.5 kHz and 7 kHz and more preferably in a band between 3 kHz and approximately 6 kHz.

In solving equation (25) for $N_1$ and $N_2$, it will be appreciated that $C_1$ , $C_2$ , $\Delta v / v|_{fluid}$ and $\Delta v / v|_{linear}$ are all determinable, and that the term $\left(v^{Stoneley} - v^{Stoneley}_{ref}\right) / v^{Stoneley}_{ref}$ which relates to the acoustoelastic coefficients of the formation is determinable. In particular, since the Stoneley wave solution does not have any azimuthal dependence, $J_1$, $J_2$, $J_3$ and $J_4$ of

equations (26) and (27) can be expressed in terms of line integrals as set forth in Appendix A hereto. Using the line integrals which utilize the shear and compressional wave velocities of the formation, $C_1$ and $C_2$ are found. Also, the portion of the fractional change in the Stoneley dispersion caused by an increase in the borehole pressure can be calculated from the linear constants of the formation in the ambient state according to:

$$\Delta v / v\big|_{linear} = J_4 / 2\omega_m^2 J_N \tag{32}$$

where the integral $J_4$ is expressed as a sum of two terms $J_{41}$ and $J_{42}$ which are likewise defined in Appendix A hereto. Further, the portion of the fractional change in the Stoneley dispersion caused by an increase in the borehole pressure that can be calculated from the known borehole fluid nonlinearity in the ambient state is given by:

$$\Delta v / v\big|_{fluid} = J_5 / J_N \tag{33}$$

where $J_5$ is expressed as an integral as set forth in Appendix A hereto. Thus, by measuring the fractional change in the Stoneley velocity $\left(\left(v^{Stoneley} - v_{ref}^{Stoneley}\right) / v_{ref}^{Stoneley}\right)$, (which relates to the acoustoelastic coefficients) the only unknowns in equation (25) are $N_1$ and $N_2$. However, a solution for $N_1$ and $N_2$ is possible by utilizing equation (25) at a plurality of discrete frequencies and conducting a multi-frequency inversion as described in more detail hereinafter.

The sensitivity of the two formation nonlinear constants $N_1$ and $N_2$ to the flexural dispersion caused by an increase in the borehole pressure is likewise obtained from equation (9). Substituting the linear (second-order) as well as nonlinear (third-order) elastic constants for an isotropic medium subject to the biasing stresses caused by an increase in the borehole pressure together with the flexural wave solution $u^m$ $(m=1)$ into equation (9), the change of phase velocity at a given frequency can be expressed as:

$$\left(v^{flexural} - v_{ref}^{flexural}\right) / v_{ref}^{flexural} = D_1 N_1 + D_2 N_2 + \Delta v / v\big|_{fluid} + \Delta v / v\big|_{linear} \tag{34}$$

Where

$$D_1 = \frac{1}{2\omega_m^2 I_N}\left(I_1 - \frac{I_3}{2}\right) \tag{35}$$

$$D_2 = \frac{1}{2\omega_m^2 I_N}\left(2I_2 + \frac{I_3}{2}\right) \tag{36}$$

$$N_1 = \frac{c_{123} - c_{112}}{2c_{66}} \tag{37}$$

$$N_2 = \frac{c_{112} - c_{111}}{4c_{66}} \tag{38}$$

In solving equation (34) for $N_1$ and $N_2$, it will be appreciated that $D_1$, $D_2$, $\Delta v/v|_{fluid}$ and $\Delta v/v|_{linear}$ are all determinable, and that the term $\left(v^{flexural} - v_{ref}^{flexural}\right)/v_{ref}^{flexural}$ which relates to the acoustoelastic coefficients of the formation is determinable. In particular, it will be appreciated that since the flexural wave solution exhibits azimuthal dependence, $I_1$, $I_2$, $I_3$ and $I_N$ can be expressed in terms of surface integrals as set forth in Appendix A hereto. In addition, the portion of the fractional change in the flexural dispersion caused by an increase in the borehole pressure that can be calculated from the linear constants of the formation in the ambient state is given by $\Delta v/v|_{linear} = I_4/I_N$ where the integral $I_4$ is expressed by:

$$I_4 = a^2\Delta P\left(I_{41} + I_{42}\right) \tag{39}$$

where $I_{41}$ and $I_{42}$ are defined in Appendix A hereto. Similarly, the portion of the fractional change in the flexural dispersion caused by an increase in the borehole pressure that can be calculated from the known borehole fluid nonlinearity in the ambient state is given by $\Delta v/v|_{fluid} = I_5/I_N$ where the integral $I_5$ is defined in Appendix A hereto. Thus, by measuring the fractional change in the flexural velocity ($\left(v^{flexural} - v_{ref}^{flexural}\right)/v_{ref}^{flexural}$) (which relates to the acoustoelastic coefficients) the only unknowns in equation (34) are $N_1$ and $N_2$. However, as was the case with the Stoneley dispersion, the solution for $N_1$ and $N_2$ is possible by utilizing equation (34) at a plurality of discrete frequencies and conducting a multi-frequency inversion as described in more detail hereinafter.

It will be appreciated from equations (35) and (36) that $D_1$ and $D_2$ are frequency dependent, and their frequency dependence for a unit increase in borehole pressure (i.e., $\Delta P$ = -lN/m) is shown in graphical form in Fig. 8. Here it is clear that the magnitude of $D_1$ and $D_2$ are essentially zero from 0 to 3 kHz. This implies that the inversion for the nonlinear constants $N_1$ and $N_2$ of the formation from the fractional changes-in the flexural wave velocities in this frequency band would result in substantially large errors for their estimated values. A preferred frequency band for the inversion of $N_1$ and $N_2$ is one where both the values of $D_1$ and $D_2$ are significantly large. However, it is known in the art that a conventional sonic tool with dipole sources and receivers placed on the borehole axis is not very efficient in recording flexural wave signals at frequencies either below about 1 kHz or much higher than about 10 kHz. (See Sinha, B.K. et al., "Borehole Flexural Modes in Anisotropic Formations", Geophysics, Vol. 59, No. 7, pp. 1037-1052 (July 1994)). With this

information, as well as the sensitivity analysis of Fig. 8, an optimal frequency band for processing the flexural wave velocity changes caused by a borehole pressurization approximately 6 kHz, and more preferably between 4.0 kHz and 5.75 kHz.

As suggested above, estimation of the formation nonlinear constants may be carried out from a multi-frequency inversion of the Stoneley and/or flexural wave velocity dispersions caused by a borehole pressure increase. Assuming that fractional changes in the Stoneley wave velocities at two frequencies $f_l$ and $f_2$ are available for a borehole pressure increase aP, the inversion process is formulated as $AX = B$ according to equations (4) - 6) set forth above:

$$A = \begin{vmatrix} c_1^{f1} c_2^{f1} \\ c_1^{f2} c_2^{f2} \end{vmatrix} \qquad (40)$$

$$X = \begin{vmatrix} N_1 \Delta P \\ N_2 \Delta P \end{vmatrix} \qquad (41)$$

$$B = \begin{vmatrix} (\Delta v/v|_{Stoneley} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f1} \\ (\Delta v/v|_{Stoneley} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f2} \end{vmatrix} \qquad (42)$$

It should be appreciated that although equations (40) - (42) relate to a two frequency inversion, three or more frequencies can be utilized in the inversion.

Table II below contains the input data of the Stoneley velocity differences ($\Delta P$ = 500 psi) at four frequencies which are grouped in pairs, and the nonlinear constant parameters of the second formation are calculated according to equations (25) and expanded equations (40) - (42). The actual values of the formation nonlinear constant parameters are shown in the parentheses.

TABLE II

Multi-frequency inversion of Stoneley Dispersion

| f kHz | $\Delta v/v|_{Stoneley}$ | $B$ | $N_1$ | $N_2$ |
|---|---|---|---|---|
| 1.688 | 0.0168 | 0.0149 | 582 | 977.5 |

| | | | | |
|---|---|---|---|---|
| 2.180 | 0.0219 | 0.0201 | (582) | (979) |
| 2.537 | 0.0255 | 0.0237 | 582 | 979 |
| 3.574 | 0.0347 | 0.0331 | | |

It is seen that the values of the nonlinear parameters $N_1$ and $N_2$ calculated from the above equations based on obtained velocity information for both pairs of frequency data agree closely with actual values for the formation.

As with the Stoneley dispersion, changes in the flexural dispersions caused by borehole pressurization can be subjected to a multi-frequency inversion to estimate the nonlinear constants of the same formation. However, the flexural dispersion data is preferably taken at a moderately high frequency band (approximately 3 to 4 KHz), as the low frequency flexural dispersion data exhibit negligibly small acoustoelastic effects. Thus, Table III below contains the input data of the flexural velocity differences at four frequencies which are grouped in pairs, and the nonlinear constant parameters of the second formation are calculated according to equation (34) and expanded equations (40) - (42), with the matrix of equation (40) being slightly modified to account for "*D*" values instead of "*C*" values.

TABLE III

Multi-frequency Inversion of Flexural Dispersion

| f kHz | $\Delta v / v\vert_{flexural}$ | $B$ | $N_1$ | $N_2$ |
|---|---|---|---|---|
| 3.412 | 0.0155 | 0.0153 | 582 | 980 |
| 3.995 | 0.0257 | 0.0253 | (582) | (979) |
| 4.054 | 0.0266 | 0.0263 | 586.6 | 978.6 |
| 1.404 | 0.0322 | 0.0318 | | |

Again, it is seen that the calculated values for the nonlinear parameters $N_1$ and $N_2$ for both pairs of frequency data agree closely with the actual values for the formation.

While nonlinear parameters $N_1$ and $N_2$ by themselves provide additional useful information regarding the formation, the parameters can also be used to help determine the type and strength of the rock formation which is being investigated, as well as the quantitative stress in the formation. According to theory, the plane wave velocities for waves propagating along the $X_1$ direction in an isotropic medium subject to homogeneous biasing normal stresses (T) and strains (E) are obtained from the equations of motion for small dynamic fields superimposed on a bias:

$$\rho_0 V_{11}^2 = c_{11} + T_{11} + (2c_{11} + c_{111})E_{11} + c_{112}(E_{22} + E_{33}) \tag{43}$$

$$\rho_0 V_{12}^2 = c_{66} + T_{11} + c_{155}E_{11} + (2c_{66} + c_{155})E_{22} + c_{144}E_{33} \tag{44}$$

$$\rho_0 V_{13}^2 = c_{66} + T_{11} + c_{155}E_{11} + c_{144}E_{22} + (2c_{66} + c_{155})E_{33} \tag{45}$$

where $\rho_0$ is the mass density of the medium, $V_{IJ}$ denotes the plane wave velocity in the reference state for propagation along the $X_I$ direction and polarization along the $X_J$ direction. If the specimen is in the form of a rod with the uniaxial stress applied along the rod axis (i.e., stresses normal to the rod axis are assumed to be zero), stress derivatives of shear waves propagating normal to the rod axis and polarized parallel and normal to the stress direction can be approximated by:

$$\frac{\rho_0 \partial V_{12}^2}{\partial S} = \frac{(2 - N_2)c_{66}}{Y} + \frac{(N_1 + N_2)\upsilon c_{66}}{Y} \tag{46}$$

$$\frac{\rho_0 \partial V_{13}^2}{\partial S} = \frac{(\upsilon N_2 - N_2)c_{66}}{Y} + \frac{(N_2 - 2)\upsilon c_{66}}{Y} \tag{47}$$

where $N_1 = -c_{144}/c_{66}$ and $N_2 = -c_{155}/c_{66}$, $N_l$ and $N_2$ are the normalized nonlinear constants of the formation, and $\upsilon$ and $Y$ are the respectively Poisson's ratio and Young's modulus of the formation in the reference ambient state. The Poisson's ratio and Young's modulus are a function of the second order constants of the formation and can be expressed by:

$$\upsilon = \frac{c_{12}}{2(c_{12} + c_{66})} \tag{48}$$

$$Y = \frac{c_{66}(3c_{12} + 2c_{66})}{(c_{12} + c_{66})} \tag{49}$$

On the other hand, if the specimen is long along the propagation direction, a plane strain approximation normal to the propagation direction $X_1$ is an appropriate assumption. In this case, when the specimen is subjected to a uniaxial stress $S$ along the $X_2$ direction, stress derivatives of shear waves polarized parallel and normal to the stress direction are given by:

$$\frac{\rho_0 \partial V_{12}^2}{\partial S} = (2 - N_2)\frac{(1 - \upsilon^2)c_{66}}{Y} + N_1\frac{\upsilon(1 + \upsilon)c_{66}}{Y} \tag{50}$$

$$\frac{\rho_0 \partial V_{13}^2}{\partial S} = N_1\frac{(\upsilon^2 - 1_2)c_{66}}{Y} - (2 - N_2)\frac{\upsilon(1 + \upsilon)c_{66}}{Y} \tag{51}$$

Therefore, stress derivatives of $\rho_0 V_{12}^2$ and $\rho_0 V_{13}^2$ can be readily approximated from either equations (46) and (47) or (50) and (51), and typically, the stress derivatives for a rock sample will fall between these approximations. The stress derivatives are functions of the formation nonlinear constants $c_{144}$ and $c_{155}$ (via nonlinear parameters $N_l$ and $N_2$), and the

linear constants $\upsilon$ and $Y$, and provide two additional formation parameters which can be used in addition to parameters such as density, and p-wave and shear wave velocities to identify the lithology of the formation. As set forth above, the nonlinear constants can be estimated from the inversion of the changes in the Stoneley or flexural dispersions over an appropriate frequency band caused by a known change in the borehole pressure.

It should be appreciated that the stress derivatives set forth in equations (46), (47), (50) and (51) may be viewed as the slopes of curves which relate the square of the plane wave velocities $V_{12}$ and $V_{13}$ to the uniaxial stress $S$. Thus, if it is possible to obtain different formation samples, subject them to different known confining pressures, stress them with known stresses, and measure and plot the shear (plane) wave velocities as a function of stress, a database of curves of different formations can be obtained where the stress derivatives set forth above will relate to the slope of the plotted curves.

With that in mind, experimental data was obtained for the compressional and two shear wave velocities in uniaxially stressed cylindrical rock samples of the second formation (parameters of the second formation being set forth in Table I above). The rock samples were subjected to a uniaxial stress along the cylindrical axis (with no confining pressure), and the compressional and shear waves (the shear waves being polarized parallel and normal to the stress axis) were excited and detected by transducers mounted normal to the stress axis, with measurements being carried out with increasing stress until the material fractured. The results of the experiment are seen in Figures 9 and 10, with Figure 9 being a plot of the plane wave velocities as a function of stress, and Figure 10 being a plot of the square of the plane wave velocities as a function of stress.

Figure 10 reveals that when the stress derivative (i.e. curve slope) is positive, the existing stress in the material is significantly less than the yield (i.e. failure) stress; whereas when the stress derivative is negative, the material is closer to failure. In fact, as the negative stress derivative increases in magnitude, the rock becomes closer to failure.

Substituting the estimated values of the formation nonlinear constants $N_1$ and $N_2$ obtained from the multi-frequency inversion of the Stoneley and flexural dispersions before and after borehole pressurization and as shown in Tables II and III, the stress derivatives of $\rho_0 V_{12}^2$ and $\rho_0 V_{13}^2$ for the assumed reference state given by $S$=0 were calculated from equations (46) and (47). When these stress derivatives are plotted in terms of the quantities shown in

Fig. 10, the resulting slopes are given by lines labeled $A_1$ and $B_1$ respectively in Fig. 10. On the other hand, when equations (50) and (51) are employed in the calculations of the stress derivatives, the resulting slopes are given by lines marked $A_2$ and $B_2$. The agreement between the calculated slopes is good in view of the different plane stress and plane strain assumptions as well as the experimental uncertainties in the data.

With the theory as set forth above, the preferred method of the invention for measuring nonlinear properties of an earth formation is shown in Figure 11 in a block diagram format. At step 100, with the borehole at a first known pressure (typically ambient), one or both of Stoneley and flexural waves, and preferably both shear and compressional waves generated by the borehole tool are measured by receivers or detectors of the borehole at a first location in the borehole. The various velocities of the different waves are calculated at step 102 utilizing known methods. For example, the Stoneley wave and flexural wave velocities (i.e., the velocity dispersions as a function of frequency) are calculated utilizing Prony's method, while the compressional wave velocity may be obtained from the detected compressional headwave. Then, at step 104, the borehole is pressurized to a second known pressure. At steps 106 and 108, the Stoneley and flexural waves, as well as shear and compressional waves are again generated and measured by the borehole tool at the first location, and the velocities (and velocity dispersions) of the different waves are calculated at the new pressure. Utilizing the Stoneley (or flexural) wave velocity dispersions, at 110, the acoustoelastic coefficients are calculated as a function of frequency according to $\left(V^{Stoneley} - V_{ref}^{Stoneley}\right) / V_{ref}^{Stoneley} \Delta P$ (or the equivalent flexural wave equation). The frequency dependent acoustoelastic coefficients so obtained can be used to provide a qualitative indication of the nonlinearity of the formation. The acoustoelastic coefficients can also be used as described below in determining the nonlinear constants $N_1$ and $N_2$ of the formation.

With the Stoneley wave and/or flexural wave dispersion curves, an indication of the nonlinear constants $N_1$ and $N_2$ of the formation are obtained at step 112 utilizing equations (25) (27), (30) - (33), and (40) - (42). In particular, with a knowledge of the borehole radius, the change in pressure, the linear constants of the formation (typically as determined from the shear and compressional wave velocities), and the mass density and nonlinear parameters of the fluid, and using equations (26), (27), (30) - (33), (40) - (42) and the equations in the Appendix hereto, equation (25) is solved for both $N_1$ and $N_2$. Substeps involved in solving equation (25) include, for each of at least two frequencies (preferably located in the optimal frequency bands discussed above with reference to Figs. 7 and 8),

determining a fractional change in the measured acoustic velocity at step 112a, and at step 112b subtracting from the fractional change a component generated by the borehole fluid and a component due to linear aspects of the formation to provide a frequency dependent nonlinear formation component (B). Then, utilizing an inversion process $AX = B$ at step 112c according to equations (40) - (42), values are obtained for the nonlinear parameters $N_1$ and $N_2$. It should be appreciated that in lieu of utilizing the inversion process, equation (25) can be solved at two or more different frequencies to yield two or more values for frequency dependent nonlinear formation components which are each equal to $C_1N_1+C_2N_@$. By substituting determinable values for $C_1$ and $C_2$ as a function of frequency, the equations can be solved simultaneously for the two unknowns $N_1$ and $N_2$ which are frequency independent. Regardless, once obtained, parameters $N_1$ and $N_2$ may then be used at step 114 to find the third-order elastic constants $c_{155}$ and $c_{144}$ according to equations $N_1 = -c_{144}/c_{66}$ and $N_2 = -c_{155}/c_{66}$.

If both sides of equation (25) are divided by the change in pressure ($\Delta P$), the left side of equation (25) evolves into the acoustoelastic coefficients of the formation as discussed above with reference to step 110. Thus, if desired, the acoustoelastic coefficients of the formation can actually be utilized in the inversion process or in a system of simultaneous equations to find the indication of the nonlinear constants $N_1$ and $N_2$ and two of the three third-order elastic constants therefrom.

The nonlinear constants of the formation $N_1$ and $N_2$ provide an excellent indication of the relative consolidation of the formation, as well as formation strength. It is also known that a crossplot of rock linear elastic constants expressed in terms of the compressional and shear velocities together with the mass density helps in identifying the rock lithology or mineralogy (see e.g., Pickett, G.R., "Acoustic Character Logs and Their Application in Formation Evaluation", Journal of Petroleum Technology, Vol. 15 p.650 (1963). This crossplot, also known as Pickett's crossplot helps in the identification of rock lithology (e.g., identification of rock type as limestone, dolomite, or clean sandstone, etc.) in terms of a constant ratio of compressional wave velocity to shear wave velocity of different magnitudes. However, in more poorly consolidated rocks, the data tend to diverge from a constant ratio resulting in a difficult interpretation of rock lithology. The nonlinear constants $N_1$ and $N_2$ add another dimension to such crossplots which serves to differentiate rock material properties on a finer scale. In particular, nonlinear elastic parameters are much more sensitive to the changes in the material structure and initial stress than conventionally

measured linear elastic moduli. (See, e.g. Yu, Irina et al., "Comparison of Linear and Nonlinear Elastic Moduli for Rocks by Use of a Granular Medium Model", Abstract for 127th Meeting of Acoustical Society of America, Cambridge, Massachusetts June 6-10, 1994).

According to the invention, the determination of the nonlinear constants $N_1$ and $N_2$ of the formation can be further utilized in conjunction with a database of experimental data in order to determine the stress in the formation, the strength of the formation, and therefrom, the amount of additional stress required to fracture the formation. In particular, at step 120, a preliminary determination of formation lithology is made. The determination can be made from previous knowledge regarding the formation obtained from formation cores, or by neutron type logging tools, gamma ray type logging tools, etc. A determination of formation lithology may also be made utilizing determinations of density, shear wave speeds, compressional wave speed, and other information (including the nonlinear constant values $N_1$ and $N_2$). (See, e.g. Castagna, J.P. et al., "Relationships Between Compressional-Wave and Shear-Wave Velocities in Elastic Silicate Rocks", Geophysics Vol. 50, p. 571 (1984); Han, D.H., "Effects of Porosity and Clay Content on Wave Velocities in Sandstones", Geophysics, Vol. 51, p. 2093 (1986); Kowallis, B. et al., "Velocity-Porosity-Clay Content: Systematics of Poorly Consolidated Sandstones", J. Geophysical Research Vol,89, p.10355 (1984). Then, at step 122, the nonlinear constants $N_1$ and $N_2$ are used in conjunction with a knowledge of the linear constants of the formation in accord with equations (46) - (51) in order to provide indications of the stress derivatives of $\rho_0 V_{12}^2$ and $\rho_0 V_{13}^2$. At step 124, the values of the stress derivatives of $\rho_0 V_{12}^2$ and $\rho_0 V_{13}^2$ (i.e. the slopes) are used to define tangents to database plots of the square of the plane wave velocities as a function of stress (such as in Fig. 10) of known formations of the same or similar lithology at the appropriate confining pressure of the formation. Preferably, the database plots include plots which relate the square of the shear velocities to stress in different rock samples (lithologies) at different confining pressures, and the slope value information is applied to an appropriate database plot. Based on the particular plots and the values of the indications of the stress derivatives, the actual stress in the formation is determined at step 126, as the slope of the curve defines where along the curve the formation stress is located. In addition, because each database plot is obtained by taking a formation sample and stressing it until it fails, the strength of the formation is known. By subtracting at step 128 the actual stress in the formation from the strength of the formation, a determination is made as to the amount of additional stress which would be required to fracture the formation.

Different techniques for matching the value of the stress derivatives to the slope of the database curves (plots) at step 124 can be utilized. A preferred technique is to take each curve, select, e.g., four local data points of the curve, fit a cubic polynomial to the four local data points, and take the derivative of the polynomial to obtain a slope at that location of the curve. The slope and an associated formation stress value are then stored. This process is repeated for additional points of the same curve (overlapping the points if desired), until the curve is represented by a plurality of different slopes with associated different formation stress values. For each lithology, different curves are obtained to represent the situation at different confining pressures. Thus, in matching the stress derivatives to the slope of the database curves at step 124, a determination of the borehole depth is first made in order to determine the confining pressure in the formation. Based on that confining pressure and a knowledge of the lithology, the appropriate curve represented by the plurality of different slopes with associated formation stress values is chosen from the database. Then, the determined in situ stress derivative value is matched to one or more of the stored slope values for the appropriate curve and the actual stress in the formation is taken as the stress associated with that stored slope value(s) or interpolated therefrom at step 126.

It will be appreciated by those skilled in the art, that one or more logs of values of the acoustoelastic coefficients, the nonlinear parameters such as $N_1$ and $N_2$, the rock strength, the stress, and the additional stress required to fracture the formation can be made by conducting the method of the invention at different locations in the borehole.

The preferred system 200 for carrying out the method of the invention is seen in Fig. 12. The system 200 of the invention preferably includes a borehole tool 220, a borehole pressurizing means 250, and a processing means 290. The borehole tool 220 is seen in Fig. 12 suspended in a borehole 201 by means of a wireline 203 and a winch 205 as is well known in the art. The preferred borehole tool 220 is a tool such as the DSI tool of Schlumberger (DSI being a trademark of Schlumberger) which includes a plurality of acoustic detectors 222, and one or both of a monopole source 224 and a dipole source 226. The monopole source 224 provides a Stoneley wave as well as a compressional headwave in all formations, and also provides a shear headwave in fast formations. The dipole source 226, on the other hand provides a flexural wave, and the shear wave arrival time can be found as the low frequency limit of the flexural wave dispersion arrivals. Thus, in accord with the invention, it is possible in fast formations to use only a monopole source, as the

Stoneley wave can be used to find the acoustoelastic coefficients, while the Stoneley wave information In conjunction with the shear wave and compressional waves information can be used to find the nonlinear parameters $N_l$ and $N_2$, as well as the actual stress in the formation and the amount of additional stress which would be required to fracture the formation. On the other hand, in slow formations, use of both the monopole source and dipole source is preferred as the low frequency limit of the flexural wave dispersion curve can be used to provide the shear wave speed. Where both monopole sources 224 and dipole sources 226 are used, they are preferably pulsed at different times. In addition, where both monopole 224 and dipole sources 226 are used, the acoustic detectors 222 should include both monopole and dipole detectors, or detectors such as four hydrophones placed ninety degrees apart from each other and capable of working in both dipole and monopole modes. In the dipole detector mode, signals from diametrically opposite hydrophones of the four hydrophone arrangement are subtracted to yield the pressure gradient at the borehole axis associated with borehole flexural waves, while in the monopole detector mode, the signals from the diametrically opposite hydrophones are added to yield the average pressure at the borehole axis associated with Stoneley wave propagation along the borehole axis.

The borehole tool 220 also preferably includes a downhole processing means 230, packer means 240, and fluid injection means 245. The downhole processing means typically takes the form of a microprocessor and associdted circuitry which is coupled to the detectors 222 of the borehole tool 220. The downhole processing means obtains voltage information relating to pressures seen by the detector, and processes the information for transmission uphole via the wireline 203. If desired, the downhole processing means can determine the shear and compressional wave velocities, as well as processing the flexural or Stoneley wave information via Prony's method. Typically, however, the heavy mathematical processing of Prony's method is accomplished uphole by the uphole processing means 290 such as a computer which is coupled to the wireline 203.

The packer means 240 of borehole tool 220 preferably comprises first and second inflatable packers located on opposite ends of the borehole tool 220 such that the sources and detectors of the borehole 220 are located between the packers. The packers are preferably arranged so that they can seal with the borehole wall and thereby seal off a portion of the borehole. The fluid injection means 245 includes a fluid storage means 247, and pump means 249 for injecting the fluid into the borehole. When the packers are inflated and in sealed relationship with the borehole wall, the borehole (and formation) between the

packers can be pressurized by injecting fluid into the borehole. With the borehole being pressurized, the monopole and/or dipole sources are energized so that data is obtained for processing in accordance with the method invention.

It will be appreciated that the uphole processing means 290 processes the information received from the downhole tool 220 via the wireline 203 according to the method of the invention (with steps 102, and 108 - 128 of Fig. 11 providing an extremely high level flowchart of the programming of the uphole processing means 290). In particular, utilizing Prony's method, the velocities of the received Stoneley or flexural waves are determined by the processing means as a function of frequency for the borehole in at least two different states of pressurization (unpressurized being one of the possible pressurization states). The compressional wave velocity and shear wave velocity are also determined according to known techniques. The processing means 290 then processes the Stoneley or flexural wave information to determine acoustoelastic coefficients according to the method invention set forth above. Also, using either the acoustoelastic coefficients or the fractional changes in the measured acoustic velocities at a plurality of frequencies, in conjunction with other information as described with reference to Fig. 11, the processing means 290 provides nonlinear parameters of the formation. Further, using database information and the nonlinear parameter information, and as described in Fig. 11, the processing means 290 can determine the stress in the formation, the strength of the formation, and therefrom, the amount of additional stress required to fracture the formation.

There have been described and illustrated herein systems and methods for measuring nonlinear properties of an earth formation utilizing a sonic borehole tool. While particular embodiments have been described. it is not intended that the invention be limited thereto, as it is intended that the invention be as broad in scope as the art will allow and that the specification be read likewise. Thus, while a particular borehole tool embodiment was described with packers and fluid injection means, it will be appreciated that other means for pressurizing the borehole could be utillzed. For example, rather than providing the borehole tool with packers and fluid injection means, a packer type device can be located on a well head in order to pressurize the entire borehole. Also, while the borehole tool was described as a wireline type tool, it will be appreciated that the invention can be embodied in measurement-while-drilling (MWD) type tools. Further, while specific equations were utilized in providing a theoretical basis for the invention, it will be appreciated that the invention is not tied exactly thereto, and that other acoustoelastic coefficients and nonlinear parameters relating to aspects of the formation can be obtained by the method of the

invention. Likewise, while the invention was described primarily in terms of utilizing a Stoneley wave with the equations directed primarily to Stoneley waves, it will be appreciated that flexural waves can be utilized to obtain the same results. It will therefore be appreciated by those skilled in the art that yet other modifications could be made to the provided invention without deviating from its spirit and scope as so claimed.

## APPENDIX A

$$J_1 = a^2 \Delta P \int_a^\infty \frac{dr}{r}\left[\left(u_{r,r} - \frac{u_r}{r}\right)u^{*}_{z,z} + u_{z,z}\left(u^{*}_{r,r} - \frac{u^{*}_r}{r}\right)\right]$$

$$J_2 = a^2 \Delta P \int_a^\infty \frac{dr}{r}\left[u_{r,r}u^{*}_{r,r} - \frac{u_r}{r}\frac{u^{*}_r}{r}\right]$$

$$J_3 = a^2 \Delta P \int_a^\infty \frac{dr}{r}\left[\left(u_{r,z} + u_{z,r}\right)\left(u^{*}_{r,z} + u^{*}_{z,r}\right)\right]$$

$$J_N = \int_0^a r dr\left[\rho_f\left(u^f_r u^{f*}_r + u^f_z u^{f*}_z\right)\right]$$

$$+\int_a^\infty r dr\left[\rho_s\left(u_r u^{*}_r + u_z u^{*}_z\right)\right]$$

where $\rho_f$ and $\rho_s$ are the fluid and formation mass densities, respectively; ($u^f_r$ and $u^f_z$) denote the Stoneley wave solution in the borehole fluid, whereas ($u_r$ and $u_z$) are the corresponding solution in the formation.

$$J_{41} = \Delta P \int_a^\infty r dr\left[\left(u_{r,r} + \frac{u_r}{r}\right)u^{*}_{z,z} + \left(u_{z,z} + \frac{u_r}{r}\right)u^{*}_{r,r}\right]$$

$$+\Delta P \int_a^\infty r dr\left[\left(u_{z,z} + u_{r,r}\right)\frac{u^{*}_r}{r} - u_{z,r}u^{*}_{r,z} - u_{r,z}u^{*}_{z,r}\right]$$

$$J_{42} = \frac{c_{12}}{c_{66}} \Delta P \int_a^\infty \frac{dr}{r}\left[\left(\frac{u_r}{r} - u_{r,r}\right)u^{*}_{z,z} + u_{z,z}\frac{u^{*}_r}{r} - u_{z,z}u^{*}_{r,r}\right]$$

$$+\frac{\left(2c_{12} + 5c_{66}\right)}{c_{66}} \Delta P \int_a^\infty \frac{dr}{r}\left[\frac{u_r}{r}\frac{u^{*}_r}{r} - u_{r,r}u^{*}_{r,r}\right]$$

$$-\Delta P \int_a^\infty \frac{dr}{r}\left[\left(u_{z,r} + u_{r,z}\right)\left(u^{*}_{z,r} + u^{*}_{r,z}\right) - u_{z,r}u^{*}_{z,r}\right]$$

$$J_5 = \frac{\Delta A}{2\omega_m^2}\int_0^a r\,dr\left[\left(u^f_{z,z} + u^f_{r,r} + \frac{u^f_r}{r}\right)\left(u^{f*}_{z,z} + u^{f*}_{r,r} + \frac{u^{f*}_r}{r}\right)\right]$$
$$-\frac{\Delta\rho_f}{2}\int_a^\infty \frac{dr}{r}\left[u^f_z u^{f*}_z + u^f_r u^{f*}_r\right]$$

where the changes in the fluid bulk modulus $\Delta A$, and mass density $\rho_f$ caused by an increase in the fluid pressure are calculated from the expressions

$$\Delta A = \left(1 + \frac{B}{A}\right)\Delta P$$

$$\Delta\rho_f = \frac{\rho_f}{A}\Delta P$$

$$I_1 = a^2\Delta P\int_0^{2\pi} d\phi\int_a^\infty \frac{dr}{r}\left[\left(u_{r,r} - \frac{u_r}{r} - \frac{u_{\phi,\phi}}{r}\right)u^*_{z,z} + u_{z,z}\left(u^*_{r,r} - \frac{u^*_r}{r} - \frac{u^*_{\phi,\phi}}{r}\right)\right]$$

$$I_2 = a^2\Delta P\int_0^{2\pi} d\phi\int_a^\infty \frac{dr}{r}\left[u_{r,r}u^*_{r,r} - \left(\frac{u_r}{r} + \frac{u_{\phi,\phi}}{r}\right)\left(\frac{u^*_r}{r} + \frac{u^*_{\phi,\phi}}{r}\right)\right]$$

$$I_3 = a^2\Delta P\int_0^{2\pi} d\phi\int_a^\infty \frac{dr}{r}\left[\left(u_{r,z} + u_{z,r}\right)\left(u^*_{r,z} + u^*_{z,r}\right) - \left(\frac{u_{z,\phi}}{r} + u_{\phi,z}\right)\left(\frac{u^*_{z,\phi}}{r} + u^*_{\phi,z}\right)\right]$$

$$I_N = \int_0^{2\pi} d\phi\int_0^a r\,dr\left[\rho_f\left(u^f_r u^{f*}_r + u^f_\phi u^{f*}_\phi + u^f_z u^{f*}_z\right)\right]$$
$$+\int_0^{2\pi} d\phi\int_a^\infty r\,dr\left[\rho_s\left(u_r u^*_r + u_\phi u^*_\phi + u_z u^*_z\right)\right]$$

where $\rho_f$ and $\rho_s$ are the fluid and formation mass densities, respectively; ($u^f_r$, $u^f_\phi$ and $u^f_z$) denote the flexural wave solution in the borehole fluid, whereas ($u_r$, $u_\phi$ and $u_z$) are the corresponding solution in the formation.

$$I_{41} = \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \left(1 + \frac{c_{12}}{c_{66}}\right)\left(\frac{u_r}{r} + \frac{u_{\phi,\phi}}{r}\right) u^{*}_{z,z} + \left(1 - \frac{c_{12}}{c_{66}}\right)\left(u_{r,r} u^{*}_{z,z} + u_{z,z} u^{*}_{r,r}\right) \right]$$

$$+ \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \frac{r^2}{a^2}\left(\frac{u_r}{r} + \frac{u_{\phi,\phi}}{r}\right) u^{*}_{r,r} - \left(\frac{2c_{12} + 5c_{66}}{c_{66}}\right) u_{r,r} u^{*}_{r,r} \right]$$

$$+ \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \left(\frac{c_{12}}{c_{66}} + \frac{r^2}{a^2}\right) u_{z,z} + \frac{r^2}{a^2} u_{r,r} \right] \left(\frac{u^{*}_r}{r} + \frac{u^{*}_{\phi,\phi}}{r}\right)$$

$$+ \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \left(\frac{2c_{12} + 5c_{66}}{c_{66}}\right)\left(\frac{u_r}{r} + \frac{u_{\phi,\phi}}{r}\right) \right] \left(\frac{u^{*}_r}{r} + \frac{u^{*}_{\phi,\phi}}{r}\right)$$

$$I_{42} = \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \left(1 - \frac{r^2}{a^2}\right) \frac{u_{z,\phi}}{r} + u_{\phi,z} \right] u^{*}_{\phi,z}$$

$$- \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \left(1 + \frac{r^2}{a^2}\right) u_{z,r} + u_{r,z} \right] u^{*}_{r,z}$$

$$- \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ 2u_{z,r} + \left(1 + \frac{r^2}{a^2}\right) u_{r,z} \right] u^{*}_{z,r}$$

$$+ \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \left(1 - \frac{r^2}{a^2}\right) u_{\phi,z} + 2\frac{u_{z,\phi}}{r} \right] \frac{u^{*}_{z,\phi}}{r}$$

$$- \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ u_{\phi,r} - \frac{r^2}{a^2}\left(\frac{u_{r\phi}}{r} - \frac{u_{\phi}}{r}\right) \right] u^{*}_{\phi,r}$$

$$+ \int_0^{2\pi} d\phi \int_a^{\infty} \frac{dr}{r} \left[ \frac{u_{r\phi}}{r} - \frac{u_{\phi}}{r} - \frac{r^2}{a^2} u_{\phi,r} \right] \left(\frac{u^{*}_{r\phi}}{r} - \frac{u^{*}_{\phi}}{r}\right)$$

$$I_5 = \frac{\Delta A}{2\omega_m^2} \int_0^{2\pi} d\phi \int_0^{a} r\,dr \left[ \left(u^f_{z,z} + u^f_{r,r} + \frac{u^f_r}{r} + \frac{u^f_{\phi,\phi}}{r}\right)\left(u^{f*}_{z,z} + u^{f*}_{r,r} + \frac{u^{f*}_r}{r} + \frac{u^{f*}_{\phi,\phi}}{r}\right) \right]$$

$$- \frac{\Delta\rho_f}{2} \int_0^{2\pi} d\phi \int_a^{\infty} r\,dr \left[ u^f_z u^{f*}_z + u^f_r u^{f*}_r + u^f_\phi u^{f*}_\phi \right]$$

where the changes in the fluid bulk modulus $\Delta A$, and mass density $\rho_f$ caused by an increase in the fluid pressure are calculated from the expressions

$$\Delta A = \left(1 + \frac{B}{A}\right) \Delta P$$

$$\Delta\rho_f = \frac{\rho_f}{A} \Delta P$$

CLAIMS

1 A method of measuring a nonlinear property of an earth formation surrounding a borehole using a sonic logging tool comprising a monopole or dipole source and a sonic detector, the method comprising:

a) using the sonic logging tool to generate and measure a first frequency dependent velocity of a Stoneley wave or flexural wave at a location in the borehole at a first pressure;

b) pressurizing the location in the borehole to a second pressure;

c) using the sonic logging tool to generate and measure a second frequency dependent velocity of a Stoneley wave or flexural wave at the location in the borehole at the second pressure; and

d) determining the nonlinear property of the formation from the first and second frequency dependent velocities ($v$) according to $(v - v_{ref})/v_{ref}\ \Delta P$, where $\Delta P$ is the difference between the first and second pressures.

2 A method as claimed in claim 1, comprising determining a frequency dependent acoustoelastic coefficient in order to determine an indication of the nonlinear property of the formation.

3 A method as claimed in claim 1 or 2, comprising determining the fractional change in the wave velocities and subtracting from the fractional change components related to the borehole fluid and to linear aspects of the formation to provide a nonlinear formation component.

4 A method as claimed in claim 3, wherein the nolinear formation component is provided substantially according to the equations:

$$\left(v^{Stoneley} - v^{Stoneley}_{ref}\right)/\, v^{Stoneley}_{ref} = C_1 N_1 + C_2 N_2 + \Delta v / v\big|_{fluid} + \Delta v / v\big|_{linear}$$

or

$$\left(v^{flexural} - v^{flexural}_{ref}\right)/\, v^{flexural}_{ref} = D_1 N_1 + D_2 N_2 + \Delta v / v\big|_{fluid} + \Delta v / v\big|_{linear}$$

wherein $\Delta v / v\big|_{linear}$ is the component related to the linear aspects of the formation, $\Delta v / v\big|_{fluid}$ is the component related to the borehole fluid, $N_1$ and $N_2$ are nonlinear constants of the formation, $C_1$ and $C_2$ are volume integrals which are a function of frequency and are claculable in terms of a Stoneley wave solution in an abient state

of the formation, and $D_1$ and $D_2$ are volume integrals which are a function of frequency and are claculable in terms of a flexural wave solution in an abient state of the formation.

5 A method as claimed in claim 4, further comprising measuring a first shear wave velocity and a first compressional wave velocity at the first pressure and using these velocities to determine $C_1$, and measuring a second shear wave velocity and a second compressional wave velocity at the second pressure and using these velocities to determine $C_2$.

6 A method as claimed in claim 4 or 5, comprising solving the equation for $N_1$ and $N_2$ by obtaining values for $v^{Stoneley}$, $v_{ref}^{Stoneley}$, $C_1$ and $C_2$ for a first frequency ($f_1$) and a second frequency ($f_2$).

7 A method as claimed in claim 6, wherein the first or second frequency is chosen from a frequency band of 2.5 kHz to 7 kHz.

8 A method as claimed in claim 5, 6 or 7, wherein solving the equation comprises conducting a multifrequency inversion according to $AX = B$ where

$$A = \begin{vmatrix} C_1^{f1} C_2^{f1} \\ C_1^{f2} C_2^{f2} \end{vmatrix}$$

$$X = \begin{vmatrix} N_1 \Delta P \\ N_2 \Delta P \end{vmatrix}$$

$$B = \begin{vmatrix} (\Delta v/v|_{Stoneley} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f1} \\ (\Delta v/v|_{Stoneley} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f2} \end{vmatrix}$$

9 A method as claimed in claim 4, comprising solving the equation for $N_1$ and $N_2$ by obtaining values for $v^{flexural}$, $v_{ref}^{flexural}$, $D_1$ and $D_2$ for a first frequency ($f_1$) and a second frequency ($f_2$).

10 A method as claimed in claim 9, further comprising measuring a first shear wave velocity and a first compressional wave velocity at the first pressure and using these velocities to determine $D_1$, and measuring a second shear wave velocity and a second compressional wave velocity at the second pressure and using these velocities to determine $D_2$.

11 A method as claimed in claim 9, wherein the first or second frequency is chosen from a frequency band of 3.5 kHz to 6 kHz.

12 A method as claimed in claim 9, 10 or 11, wherein solving the equation comprises conducting a multifrequency inversion according to $AX = B$ where

$$A = \begin{vmatrix} D_1^{f1} D_2^{f1} \\ D_1^{f2} D_2^{f2} \end{vmatrix}$$

$$X = \begin{vmatrix} N_1 \Delta P \\ N_2 \Delta P \end{vmatrix}$$

$$B = \begin{vmatrix} (\Delta v/v|_{flexural} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f1} \\ (\Delta v/v|_{flexural} - \Delta v/v|_{linear} - \Delta v/v|_{fluid})_{f2} \end{vmatrix}.$$

13 A method as claimed in any of claims 4 to 12, further comprising obtaining a database of experimental data for a plurality of other formations relating stress to squares of shear wave velocities, and utilizing the nonlinear constants and the database of experimental data to find the stress in the formation and/or the additional stress required to fracture the formation.

14 A method as claimed in claim 13, further comprising choosing a stress-shear velocity squared curve from the database relating to a formation of substantially similar lithology to the formation surrounding the borehole, and finding a value relating to the slope of the curve in order to find a point along the curve by utilizing the nonlinear constants, Young's modulus for the formation in an ambient reference state, Poisson's ration for the formation in an ambient reference state, and values of linear parameters of the formation, the point along the curve providing an indication of stress in the formation.

15 A method as claimed in claim 14, wherein the value relating to the slope of the curve is found according to one of the equations

$$\frac{\rho_0 \partial V_{12}^2}{\partial S} = \frac{(2 - N_2)c_{66}}{Y} + \frac{(N_1 + N_2)\upsilon c_{66}}{Y}$$

$$\frac{\rho_0 \partial V_{13}^2}{\partial S} = \frac{(\upsilon N_2 - N_2)c_{66}}{Y} + \frac{(N_2 - 2)\upsilon c_{66}}{Y}$$

or

$$\frac{\rho_0 \partial V_{12}^2}{\partial S} = (2 - N_2)\frac{(1 - \upsilon^2)c_{66}}{Y} + N_1 \frac{\upsilon(1 + \upsilon)c_{66}}{Y}$$

$$\frac{\rho_0 \partial V_{13}^2}{\partial S} = N_1 \frac{(\upsilon^2 - 1_2)c_{66}}{Y} - (2 - N_2)\frac{\upsilon(1 + \upsilon)c_{66}}{Y}$$

where $V_{12}$ and $V_{13}$ are shear wave velocities in the formation having lithology substantially similar to the formation under investigation along a first direction ("1") and polaraized along perpendicular directions thereto ("2" and "3" respectively), $S$ is the uniaxial stress along one of the perpendicular directions, $Y$ is Young's modulus, and $\upsilon$ is Poisson's ratio.

16 Apparatus for measuring a nonlinear property of an earth formation surrounding a borehole, comprising, a sonic logging tool comprising a monopole or dipole source and a sonic detector, and borehole pressurizing means for changing the pressure in the borehole at the location from a first pressure to a second pressure; wherein the tool is used to generate and measure first and second second frequency dependent velocities of a Stoneley wave or flexural wave at the location in the borehole at the first and second pressures, processing means being provided for determining the nonlinear property of the formation from the first and second frequency dependent velocities ($v$) according to $(v - v_{ref})/v_{ref}\, \Delta P$, where $\Delta P$ is the difference between the first and second pressures.

17 Apparatus as claimed in claim 16, wherein the borehole pressurizing means comprises first and second inflatable packers on the borehole tool with the source and receiver located therebetween, the packers being inflatable to contact and seal with the borehole wall.

18 Apparatus as claimed in claim 17, wherein the borehole pressurizing means includes fluid storage means for storing fluid in the tool, and fluid injection means coupled to the fluid storage means for injecting the fluid into the borehole when the first and second packers are inflated.

19 Apparatus as claimed in claim 16, 17 or 18, wherein the tool generates and measures a first shear wave and a first compressional wave at the first pressure, and a second shear wave and a second compressional wave at the second pressure, the processing means using the measurements of the first and second shear waves and compressional waves to determine the nonlinear property of the formation.

20 Apparatus as claimed in any of claims 16 to 19, further comprising a database of experimental data for a plurality of other formations relating stress to squares of shear wave velocities, the processing means utilizing nonlinear constants and the database of experimental data to find the stress in the formation and/or the additional stress required to fracture the formation.

21 Apparatus as claimed in claim 20, wherein the processing means chooses a stress-shear velocity squared curve from the database relating to a formation of substantially similar lithology to the formation surrounding the borehole, and finds a value relating to the slope of the curve in order to find a point along the curve by utilizing the nonlinear constants, Young's modulus for the formation in an ambient reference state, Poisson's ratio for the formation in an ambient reference state, and values of linear parameters of the formation, the point along the curve providing an indication of stress in the formation.

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| **Patents Act 1977**<br>**Examiner's report to the Comptroller under Section 17**<br>**(The Search report)** | Application number<br>GB 9516878.7 |
| **Relevant Technical Fields**<br><br>(i) UK Cl (Ed.N) G1G-GEL, GMB<br><br>(ii) Int Cl (Ed.6) G01V-1/00, 1/48, 1/50 | Search Examiner<br>S J DAVIES<br><br>Date of completion of Search<br>9 NOVEMBER 1995 |
| **Databases** (see below)<br>(i) UK Patent Office collections of GB, EP, WO and US patent specifications.<br><br>(ii) ONLINE WPI, INSPEC | Documents considered relevant following a search in respect of Claims :-<br>1-21 |

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